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Comments on "A high resolution Mars surface gravity grid" (Górski et al. 2018, Planetary and Space Science 160, 84–106)

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Abstract

Górski et al. (2018, Planetary and Space Science 160, 84–106, doi: 10.1016/j.pss.2018.03.015) developed a high-resolution surface gravity grid for Mars. They presented a comparison with the Mars Gravity Model 2011 (MGM2011) by Hirt et al. (2012, Planetary and Space Science 67, 147–154. doi: 10.1016/j.pss.2012.02.006), which is based on similar input data, but a different computational methodology. These two models do not agree very well in their higher-frequency constituents, that is, the parts related to topography-induced gravity. In the spatial domain, the gravity accelerations from both models differ by ~20 mGal (root-mean-square) and reach amplitudes as large as ~695 mGal in places. Górski et al. (2018) suggest that some aspects of the MGM2011 modelling algorithm are "in error", but without substantiating this claim nor verifying their model. In this communication, we present results from our validation experiments using alternative gravity modelling techniques, all showing a close fit with MGM2011, while being in clear disagreement (to a level of ~40 % of the signal) with the Górski et al. (2018) model. We discuss and narrow down the origin of the mismatches to a scale-factor of square-root (2), apparently included in their high-frequency modelling. As a general conclusion, our experiments indicate that the Górski et al. (2018) model should be treated with some caution, especially over rugged topography so long as the underlying modelling techniques are not comprehensively validated.

1. Introduction

Górski et al. (2018) describe the development of high-resolution grids of gravitational acceleration vectors on the surface of Mars, hereafter abbreviated to JPL2018. Together with the Mars Gravity Model (MGM2011), developed by Hirt et al. (2012a), there are now two gridded data sets available that describe gravity accelerations ("g-values") at the topographic surface of Mars with km-level spatial resolution. As shown in Górski et al. (2018, Fig. 25), g-values from JPL2018 and MGM2011 are not identical. Figure 1 here illustrates the spatial distribution of the differences, which have a root-mean-square (RMS) value of ~20.4 mGal and frequently exceed 100 mGal, with maximum amplitudes of up to ~695 mGal. Overall, g-values

from the two models do not agree very well in many regions, especially those with rugged topography (Fig. 1). The observed discrepancies may be unacceptably large for applications requiring accurate surface g-values.

The purpose of this communication is to present results from our own validation experiments, clarifying the source of the differences, beyond the considerations made by Górski et al. (2018). We first summarise relevant similarities and differences among the two models (Sect. 2) before presenting results from validation experiments (Sect. 3), allowing us to narrow down the origin of the mismatch between the two models. The findings are discussed and some recommendations given in Sect. 4.

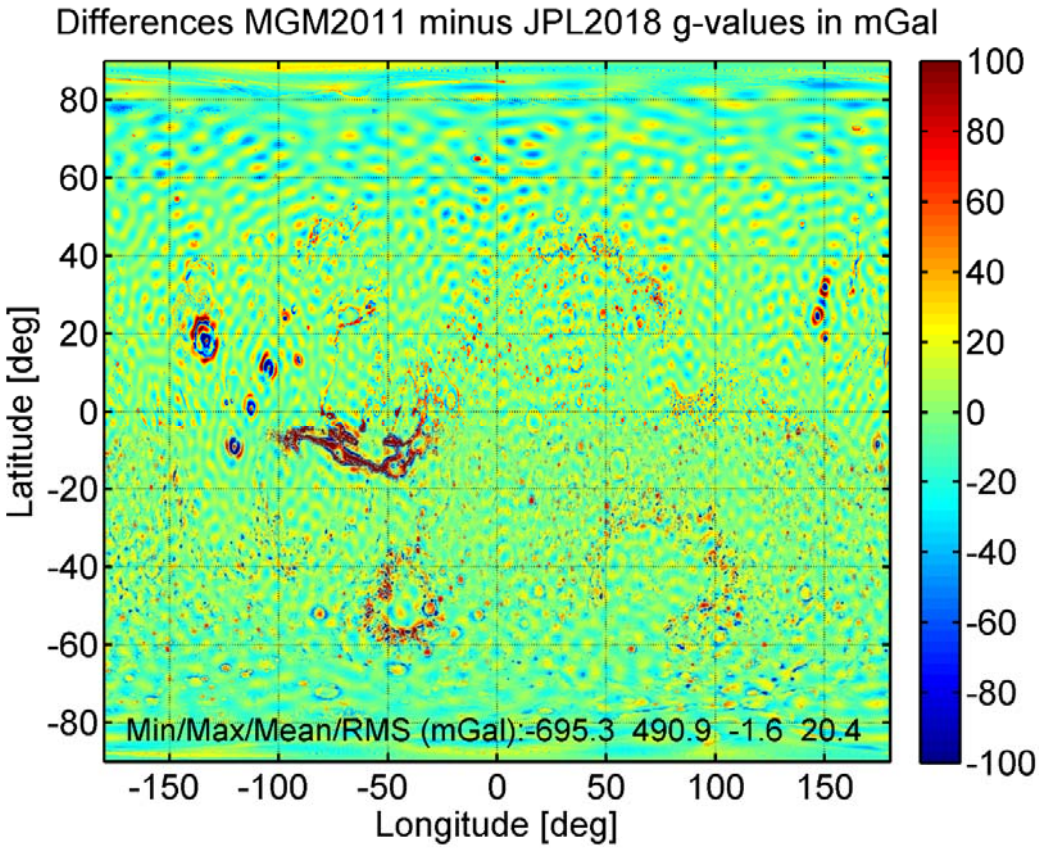

Fig. 1. Differences between MGM2011 and JPL2018 gravity accelerations.

2. Brief comparison between input data and modelling techniques

The JPL2018 and MGM2011 surface g-value models are similarly based on combinations of (1) observed gravity relying on spacecraft tracking data, coupled with (2) forward-modelled gravity using models of Martian topography together with some assumed-constant mass-density value. The first component contributes the long-wavelength part, while the second component represents short-wavelengths that cannot be sensed by spacecraft orbit analysis due to gravity attenuation with altitude. The basic idea behind both models – the use of topography-implied (TI) gravity to model the fine structure of the gravitational field

– has been used in Earth gravity modelling (e.g., Forsberg and Tscherning 1981) before being adopted in planetary sciences, to Mars (e.g., Hirt et al. 2012a, Górski et al. 2018) or other planetary bodies. The underlying assumptions of the MGM2011 forward modelling from geometry only (e.g., 3D variable density distribution and isostatic compensation) are acknowledged in Hirt et al. (2012a, section 3.3).

Both MGM2011 and JPL2018 rely on similar (though not identical) input data sets:

(a) Gravity from spacecraft tracking: MGM2011 relies on the MRO110B2 model (Konopliv et al. 2011) to spherical harmonic degree and order 85, while JPL2018 uses the updated product named MRO120D (Konopliv et al. 2016) to degree and order 80.

(b) Topographic mass models: Both MGM2011 and JPL2018 use global topography from the Mars Orbiter Laser Altimeter (MOLA) instrument at a 64 pixel per degree spatial resolution (Mission Experiment Gridded Data Record). Given the similar or identical topography data used, a reasonably good fit among the high-frequency forward-modelled gravity constituents should be expected, even when using independent techniques (e.g., Hirt et al. 2016).

The first important difference is the adopted uniform topographic mass-density: 2900 kg m^{-3} for MGM2011 and 2782 kg m^{-3} for JPL2018. However, this difference is not critical here because it is considered in our validation experiments (Sect. 2). Even without explicit consideration, the different mass-densities translate into a scale-factor of less than 5%, so cannot explain differences at the 40% level (cf. Sect. 3). As a secondary difference, JPL2018 has a higher spatial resolution (~1.73 km) versus MGM2011 (~3 km) at the equator.

The third and most significant difference concerns the numerical techniques used for the computation of g-values from the respective input data:

- MGM2011 g-values are obtained as sum of three constituents: (1) A reference ("normal") gravity field accounts for the gravitational attraction of Mars, as approximated by a rotating ellipsoid of uniform mass-density, at the 3D field points represented through MOLA topography. (2) MRO110B2 provides the long-wavelength gravity signals at the MOLA topography. (3) Short-scale gravity signals are forward-modelled based on the residual terrain modelling (RTM) technique (Forsberg and Tscherning 1981), where a high-pass filtered topographic mass model is used and cap-limited numerical integration (NI) approximates the gravity field's fine structure not captured by (2).
- In the Górski et al. (2018) approach to determine JPL2018, no reference gravity field is used. Instead of using RTM, the short-wavelength part is "*derived from convolution of appropriate Green's functions with the surface mass distribution*". This approach is based on spherical-harmonic modelling and combines internal and external topography-generated potentials. The final "*Mars gravity model is represented by a combination of the MRO120D model, truncated at harmonic degree n = 80, and the topography-derived model for harmonic degrees above n = 80*." (Górski et al. 2018).

3. Comparisons and validation experiments

Górski et al. (2018, Fig. 27) harmonically analysed the MGM2011 and JPL2018 g-value grids and found a drop in spectral energy in MGM2011 w.r.t. JPL2018 near degree 85, that is, at the transition from observed to TI gravity. Such a drop in spectral energy is absent in the spectrum of JPL2018 around degree 80 (their transition degree). Figure 2 shows our replication of the harmonic analyses as already reported by Górski et al. (2018), compare the red and blue lines. For the further validations presented here, we have developed two experimental Mars g-value grids with a methodology independent from the MGM2011 and JPL2018 methodologies, as follows:

1) A model of the external gravitational potential generated by the Martian topography was created using spherical harmonic analysis of the topography and integer powers thereof (i.e., spectral-domain gravity forward modelling; e.g., Chao and Rubincam 1989, Hirt et al. 2016) resulting in a topographic potential model (TPM). Our modelling was limited to degree 250 which is more than sufficient for our comparison of the discontinuity at degree 85. Note that the adopted degree is low enough to ensure that series divergence (which can be encountered when evaluating high-degree expansions of gravity field functionals inside the Brillouin sphere encompassing all planetary mass) is not necessarily a problem. This was verified by comparing spectral-domain with spatial-domain gravity forward modelling for Mars following the experimental concepts in Hirt and Kuhn (2017).

2) The TPM from step 1 was combined with "observed" MRO spacecraft gravity in the spectral-domain by merging the coefficients in the following two variants that match MGM2011 and JPL2018 respectively:

- Validation model 1A: MRO110B2 (N<=85) with TPM (density = 2900 kg m^{-3} and N>85)
- Validation model 1B: MRO120D (N<=80) with TPM (density = 2782 kg m^{-3} and N>80)

3) Gravity accelerations g were computed from the potential coefficients from step 2 at the MOLA 3D topography using Eq. 121 from Barthelmes (2013).

4) Harmonic analysis of the gravity acceleration grid from step 3.

This methodology avoids the use of reference gravity fields and RTM forward modelling (as in MGM2011), as well as the combination of internal/external potentials and Green's functions (as in JPL2018), so provides an independent check on the two modelling methodologies.

The g-value spectra from these independent validation models are shown in black and grey in Fig. 2, and differences among the model coefficients in dashed lines. Both validation models are in much closer agreement with MGM2011 than with JPL2018 for that part of the spectrum where TI information was used (n>85). This holds regardless of the adopted mass-density. Conversely, JPL2018 mismatches MGM2011 (as already observed by Górski et al. (2018) and replicated by us), but also with these validation models (black and red dashed lines in Fig. 2). The mismatch among JPL2018, MGM2011 and the validation models is about a factor of 1.4 or 40% (in terms of degree standard deviations) or a factor ~2 (in terms of degree variances).

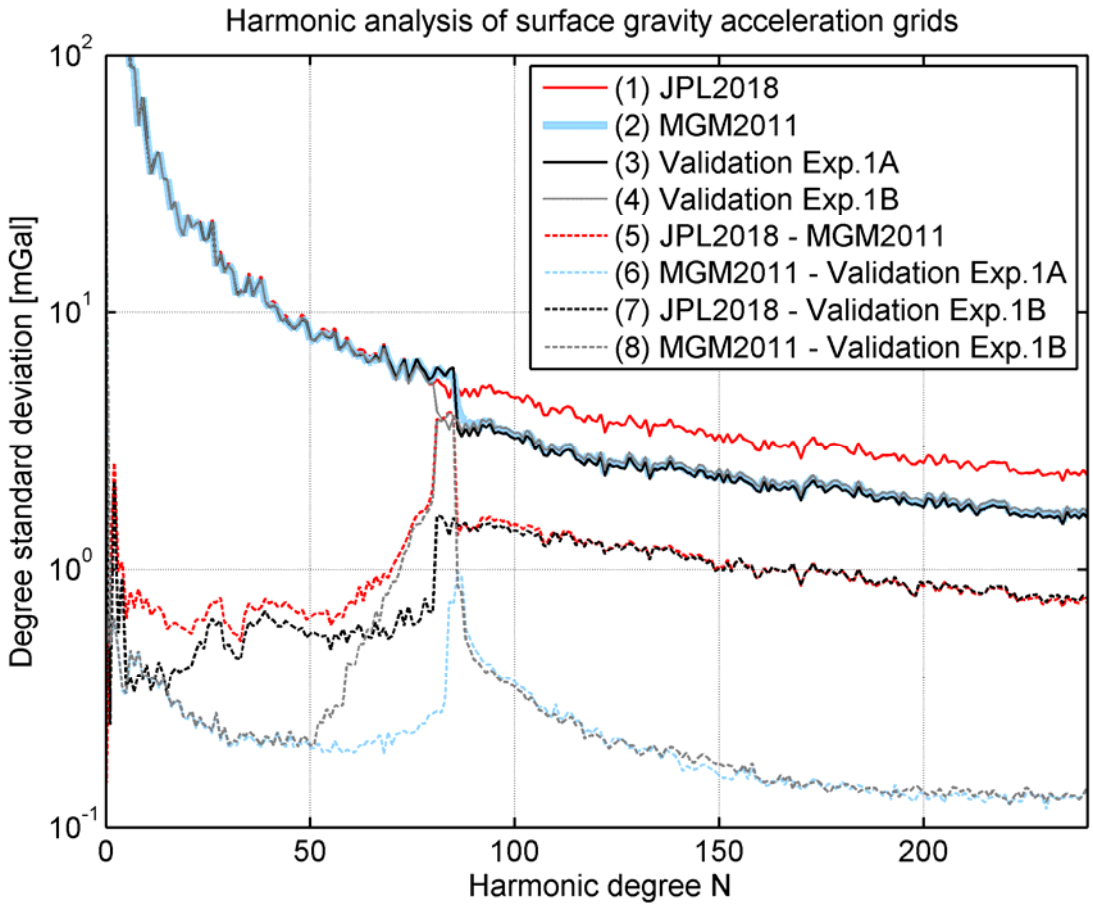

Fig. 2. Spectra of gravity accelerations from JPL2018, MGM2011 and two validation models (solid lines) and their differences (dotted lines), units in mGal.

We now focus on a comparison of the Mars gravity signal implied by the topographic masses only. By using gravity disturbances (i.e., radial derivatives of the topographic potential) instead of gravity accelerations, it is possible for us to include another independent validation data set obtained through a global NI.

In this approach, Newton's law of gravitation is evaluated in the spatial domain using a combination of spherical tesseroids and right-rectangular prisms to represent the topographic mass distribution. Evaluation points are located at the 3D MOLA topographic surface and the integration radius is 180° in each case, allowing us to recover the full TI gravity signal. The NI is distinct from spectral or RTM techniques, which is why it provides an entirely independent check.

We include the following four gravity disturbance data sets in this comparison:

(1) MGM2011 gravity disturbances, as shown in Hirt et al. (2012a, Fig. 3). At short spatial scales, MGM2011 relies on the RTM technique,

(2) Global NI of the MOLA topography at 0.05° spatial resolution using the Newtonian forward-modelling integrator by Curtin University,

(3) Spectral forward modelling, where the gravity disturbances were obtained through gravity syntheses of the TPM coefficients (see above) at the 3D MOLA topographic surface, and

(4) JPL2018, whereby gravity disturbances were obtained through a simple subtraction of the 3D normal gravity field (available from the MGM2011 model development, cf. Hirt et al. 2012a, Fig, 2b) from the JPL2018 g-value grid.

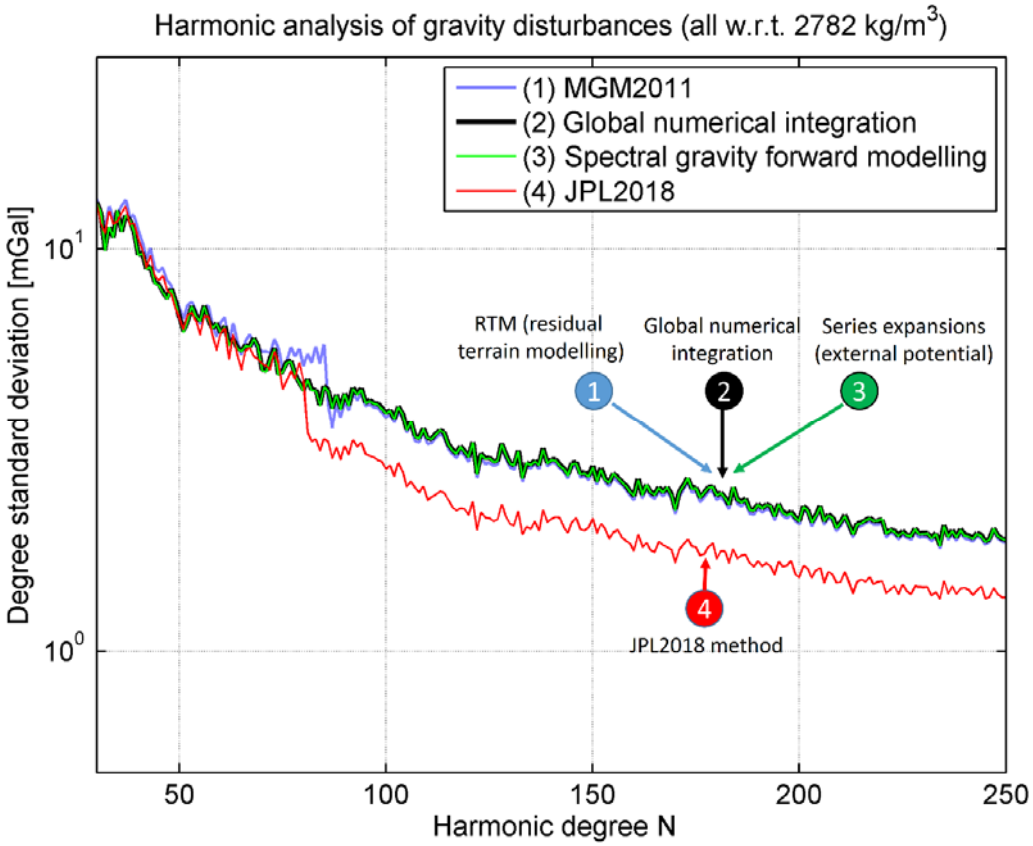

Fig. 3. Spectra of gravity disturbances from JPL2018, MGM2011 and the two validation models "NI" and "spectral gravity forward modelling". The spike visible for MGM2011 near n=85 is a result of the RTM filtering problem (see Rexer et al. 2018) together with the MRO110B2 signal structure. It diminishes when adopting MRO120D and the Rexer et al. (2018) methods for MGM2011.

For the validation data sets (2) and (3) above, the mass-density of 2782 kg m^{-3} was adopted to match that used in JPL2018, and the MGM2011 gravity disturbances were scaled to that density value for n>85. Figure 3 displays the power spectra obtained through spherical harmonic analysis of the four grids of gravity disturbances. MGM2011 (1) is seen to be in close agreement with independent techniques (2) and (3), with differences at the level of few percent of the gravity disturbance signal. JPL2018 (4) mismatches any of the other three techniques, despite all adopting the JPL2018 mass-density value. The JPL2018 TI gravity disturbance signals for n>85 appear to be too small (cf. Fig. 3). A smoother continuation/transition between the gravity spectra of the spacecraft-models to the TI components is visible for MGM2011 and the two validation data sets, but not for JPL2018.

4. Discussion and concluding remarks

Górski et al. (2018) noted the mismatch between the MGM2011 and JPL2018 g-value spectra, and we replicate it here. Without providing due evidence, however, Górski et al. (2018, p 101) made the claim: "*Perhaps the most puzzling feature of this difference is that Hirt et al. used a higher density than we did, but our gravity model has a higher variance value than theirs does. This suggests that some aspects of their algorithm are in error*", while not taking into consideration or even acknowledging the possibility that the algorithms or computations behind their own model could equally and legitimately explain the observed differences.

We believe that not only the outcomes of the validation experiments presented in Sect. 3, but also a MGM2011 replication experiment using Earth as "test laboratory" (Hirt et al. 2012b), when taken together have verified the validity of the MGM2011 modelling techniques. Also, MGM2011 is based on exactly the same modelling approach as the GGMplus g-value maps, which have been successfully validated with several million ground-truth gravity observations around the Earth (Hirt et al. 2013).

Our validation experiments show the TI component of JPL2018 to be in mismatch with any of these three well-established and widely used forward-modelling techniques (RTM, global NI and external spherical harmonics). To the best of our knowledge, the Górski et al. (2018) modelling technique has not been verified (well, at least not published in the open literature) with some independent methodology (e.g., NI, tested to construct an Earth gravity field model that would enable a validation with ground-truth gravity, or some other independent validation). In light of all this available evidence to us, we do not agree with the above claim by Górski et al. (2018) and thus defend the veracity of our MGM2011 Martian g-value model.

The more intriguing and probably less intuitive feature observed by Górski et al (2018) and shown in Fig. 2 here is the drop in spectral energy in the MGM2011 gravity acceleration spectrum (as well as in the validation models), occurring exactly at the transition from spacecraft-observed to TI gravity data. To our understanding thus far, this drop could be an indication that the gravity-from-topography values neglect Bouguer gravity signals present in the real (albeit unknown) Martian gravity field, relating to factors such as 3D density variations or isostatic compensation effects. This drop may not be present in case observed gravity and gravity-from-topography were strongly correlated (near +1) at the transition wavelength. One possible and plausible explanation for this empirically observed feature can be deduced when decomposing the spacecraft-topography-combined gravity acceleration model into the gravity effect of the rotating mass-ellipsoid and that of all contributions residual to it (topographic and all anomalous masses). The following mechanisms seem to hold:

- For harmonic degrees n>85, no mass-density anomalies are modelled because of the constant mass-density assumption in the forward modelling for any model. Thus, in case of Mars and in planar approximation, a normal gradient of ~-0.22 mGal/m (from the mass-ellipsoid) would directly

counteract a Bouguer plate effect of ~0.12 mGal/m (topographic masses) in the g-values at short scales. What can be thought of as a cancellation effect reduces the spectral power of the resulting g-values, them being the sum of both components.

- For n<=85, the models rely on spacecraft gravity observations, so contain field structures generated by all planetary masses, including the topographic masses and, importantly, all mass-density anomalies beneath the topography and the planetary interior. In case that substantial mass-density anomalies exist, the correlation between gravity and topography is lower than +1 (in the present case, around +0.7 to +0.8, see Hirt et al. 2012a, Fig. 1). As a result, the signal power of the gravity accelerations increases because of the composite effect of gravity contributions from the mass-ellipsoid, from the topography and, importantly, also from all mass-density anomalies. A cancellation effect as described above for n>85 is therefore not so pronounced.

These mechanisms seem to provide some reasonable explanation for the drop in power at n=85. Of course, the gravity acceleration spectrum of the real field may naturally not exhibit such a drop. However, it is a feature to be expected for gravity acceleration *models*, when augmenting a spacecraft model with TI gravity data at some transition wavelength where gravity and topography are not very closely correlated. Such a drop can be considered plausible for Mars (gravity-topography-correlation of around +0.8), but it cannot be expected, e.g., for the Moon, where gravity-topography-correlation is close to +1 at short spatial scales for modern GRAIL-based gravity models (e.g., Šprlák et al. 2018).

To our understanding, the JPL2018 modelling "avoided" the drop in the gravity acceleration spectrum by determining a best-fitting mass density in the "gravity acceleration space" (Górski et al. 2018, Eq. 61), which is the most likely cause for the majority of the mismatch with MGM2011 and our two independent validation models. Due to this aspect of the JPL2018 modelling technique, a smooth transition from spacecraft to TI gravity was reached (dare we say "forced") for the gravity acceleration spectrum, but not for the gravity disturbance spectrum. Any of our validation experiments provide some evidence that this choice reduces the accuracy of the JPL2018 TI gravity component.

Specifically, the mismatch between the topography-implied JPL2018 g-values and our validation models seems to be a factor of ~1.4, which so happens to be close to the ratio between the mean overall Martian and crustal density of Mars (~3933 kg m^{-3} / 2782 kg m^{-3}). A closer look at the Górski et al. (2018) modelling technique shows that gravity acceleration spectra from MRO data and topography were compared to derive a scale factor of $\sqrt{2}$ (Górski et al 2018, p 97) that was considered in their algorithm. We believe that this is a prime candidate to explain the ~40% signal strength differences between JPL2018 gravity and that of the three comparison data sets (Fig. 3).

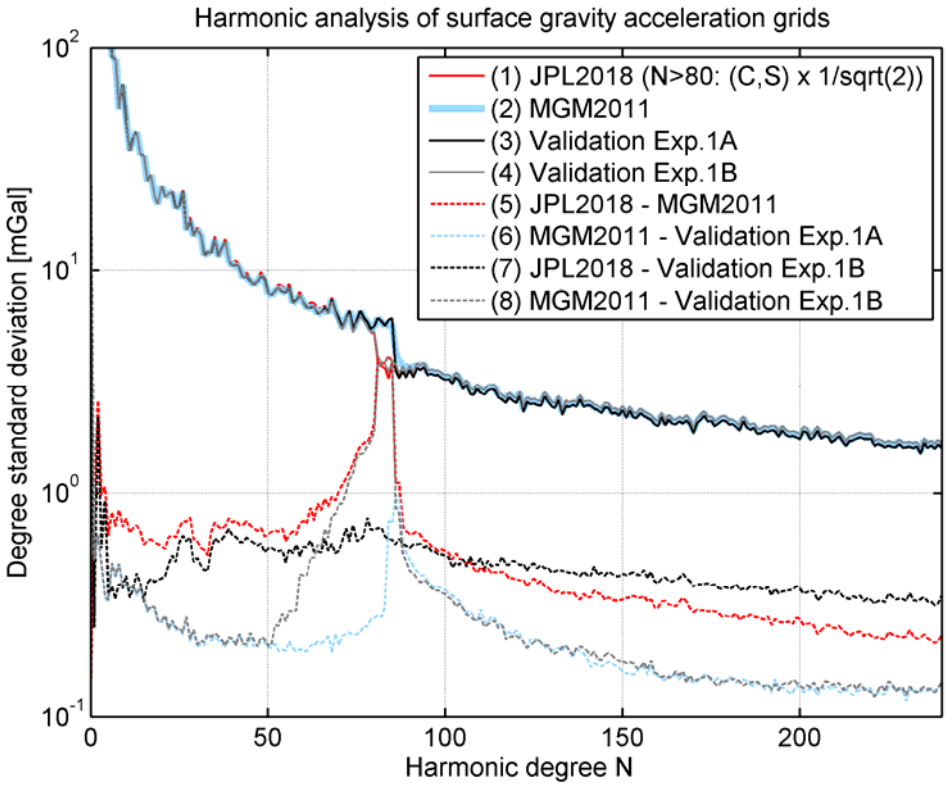

Fig. 4. As Fig. 2, but with the JPL2018 coefficients (n>80) re-scaled by a factor of $1/\sqrt{2}$

As a further test, therefore, we have applied a factor of $1/\sqrt{2}$ to the JPL2018 coefficients with n>80, yielding an improved agreement with MGM2011 and the validation models (see Fig. 4). This appears to corroborate the factor $\sqrt{2}$ to be chiefly responsible for the mismatch. It is worth noting that the determination of a best-fitting topographic mass-density is usually done by comparing potential coefficients or potential degree variances (e.g., Konopliv et al. 2014).

Finally, we willingly acknowledge that some deficiencies in the MGM2011 modelling technique have become evident over the past years, among them, e.g., a rather simplistic combination of RTM with spacecraft data (also see Fig. 3) or approximation errors in the RTM technique itself (Hirt et al. 2019). These and any further issues should be rectified when developing a next-generation Mars gravity model, e.g., by adopting the RTM technique improvements by Rexer et al. (2018). Nevertheless, these shortcomings in MGM2011 only create errors well below the ~40% mismatch with JPL2018 observed in Figs. 1, 2 and 3.

As our key conclusion, these and our other validation experiments (Hirt et al. 2012b, Hirt et al. 2013) taken together provide evidence that MGM2011 is not the main contributor to the differences w.r.t. JPL2018. While JPL2018 has been found to be in reasonable agreement with MGM2011 over non-rugged terrain in Górski et al. (2018), spurious differences are encountered over areas with rough topography (also see Fig. 1 here). Over these regions, nevertheless, we recommend that JPL2018 be treated with some caution and scepticism, until the underlying modelling algorithms have been checked and/or verified with some independent methodology.

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