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Combining EGM2008 and SRTM/DTM2006.0 residual terrain model data to improve quasigeoid computations in mountainous areas devoid of gravity data

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Abstract:

A global geopotential model, like EGM2008, is not capable of representing the high-frequency components of Earth's gravity field. This is known as the omission error. In mountainous terrain, omission errors in EGM2008, even when expanded to degree 2190, may reach amplitudes of 10 cm and more for height anomalies. The present paper proposes the utilisation of high-resolution residual terrain model (RTM) data for computing estimates of the omission error in rugged terrain. RTM elevations may be constructed as the difference between the SRTM (Shuttle Radar Topography Mission) elevation model and the spherical harmonic topographic expansion DTM2006.0. Numerical tests, carried out in the German Alps with a precise gravimetric quasigeoid model (GCG05) and GPS/levelling data as references, demonstrate that RTM-based omission error estimates improve EGM2008 height anomaly differences by 10 cm in many cases. The comparisons of EGM2008-only height anomalies and the GCG05 model showed 3.7 cm standard deviation after a bias-fit. Applying RTM omission error estimates to EGM2008 reduces the standard deviation to 1.9 cm which equates to a significant improvement rate of 47%. Using GPS/levelling data strongly corroborates these findings with improvement rates of 49% . The proposed RTM approach may be of practical value to improve quasigeoid determination in mountainous areas without sufficient regional gravity data coverage, e.g., in parts of Asia, South America or Africa. As a further application, RTM omission error estimates will allow a refined validation of global gravity field models like EGM2008 from GPS/levelling data.

Keywords: Quasigeoid determination, EGM2008, residual terrain model (RTM), omission error, commission error

1. Introduction

The EGM2008 Earth Gravitational Model, released by the US National Geospatial Agency in April 2008 (Pavlis et al. 2008; http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html), is a state-of-the-art high-degree global geopotential model (GGM) of the Earth's external gravity field. It is complete to spherical harmonic degree and order 2160 and provides some additional spherical harmonic coefficients to degree 2190. This corresponds to a spatial resolution of 5 arc minutes or ~9 km, depending on latitude.

However, quasigeoid heights (aka height anomalies) and other gravity field quantities computed solely from a GGM are always subject to the signal omission error (e.g., Gruber 2009). The omission error comprises high-frequency gravity field signals that cannot be represented by a truncated GGM spherical harmonic series expansion (e.g., Torge 2001), i.e., all gravity field features occurring at scales finer than the GGM's spatial resolution. For EGM2008 expanded to degree 2160, the global average omission error of height anomalies is estimated to be about 4 cm (Jekeli et al. 2009), but this can be larger in mountainous terrain.

If height anomalies only from EGM2008 are used (e.g., as a height reference surface, cf. Benner et al. 2009), then the impact of the omission error is implicitly accepted. In low-lying terrain, this might be acceptable because smaller effects of signal omission may generally be expected compared to mountainous areas. This is because the Earth's topography is a main source of high-frequency gravity field signals (e.g., Forsberg 1984). While GGM-only height anomalies may be acceptable for a number of applications in geosciences, many geodetic applications (particularly GPS levelling) require information on the high-frequency quasigeoid signals.

There are (at least) two ways to model the high-frequency signals not provided by a truncated GGM series expansion, thus reducing the signal omission error:

(1) The most commonly used methodology is the remove-restore approach known from regional geoid/quasigeoid modelling via Stokes's integral. In brief, GGM-implied gravity anomalies are subtracted from a set of (regionally distributed) terrestrial gravity observations, yielding residual gravity anomalies. These residual gravity anomalies are transformed to residual height anomalies using Stokes's formula and added to GGM-implied long-wavelength height anomalies.

(2) In medium-elevated and rugged terrain, residual terrain model (RTM) data (Forsberg and Tscherning 1981, Forsberg 1984; also see Forsberg 1985) may be used for source-modelling high-frequency gravity field signals. In RTM modelling, a digital terrain model (DTM) – representing Earth's topography by prisms – is referred to a long-wavelength reference surface. This step removes the low-frequency components from the DTM already implied by the GGM (cf. Forsberg 1984, 1994). The transformation of RTM elevations to residual (or RTM) height anomalies is accomplished using forward-modelling gravitational potential formulas for prisms (cf. Nagy et al. 2000).

In regions with sufficient terrestrial gravity data coverage, variant (1) generally allows more accurate modelling of the gravity field's fine structure than the RTM approach alone. This is because the RTM technique (variant 2) is usually based on simplifications of the distribution of mass-densities inside the topography. Often, a standard rock density is uniformly used for the complete RTM, thus neglecting the impact of any local density variations (cf. Hirt 2010). In regions with insufficient distribution or scarce availability of gravity data, the local gravimetric refinement of the quasigeoid through variant 1 is of limited use or sometimes even impossible. Particularly in mountainous terrain, variant 2 represents a simple and promising alternative.

In this paper, we investigate the RTM approach for modelling the high-frequency gravity field in mountainous regions in order to improve quasigeoid information from EGM2008 alone (Sect. 2). The RTM data is constructed from two freely accessible data sources: Shuttle Radar Topography Mission (SRTM) elevations (Farr et al. 2007) and the long-wavelength DTM2006.0 spherical harmonic model of Earth's topography (Pavlis et al. 2007). We describe the transformation of RTM elevations to height anomalies, analyse the spatial extent over which RTM elevations must be evaluated, and discuss the role of unmodelled local mass-density anomalies (Sect. 3).

Based on numerical tests in the German Alps where the German Combined Quasigeoid GCG05 (Liebsch et al. 2006) and a GPS/levelling data set are available for comparison (Sect. 4), we demonstrate that the RTM approach is capable of improving EGM2008 in mountainous terrain (Sect. 5). Particular focus is placed on analysing the role of the spherical harmonic degree used for combining EGM2008 and RTM. We consider our approach of value in all mountainous regions where the Stokes-based modelling of the high-frequency gravity field components may not be feasible (Sect. 6). The present work is complementary to Hirt (2010), which showed that RTM data significantly improves vertical deflections computed from EGM2008.

2. EGM2008 height anomalies

In order to compute height anomalies $\zeta^{EGM2008}$ from the set of EGM2008 fully-normalised spherical harmonic coefficients $\overline{C}_{nm}$, $\overline{S}_{nm}$ (Pavlis et al. 2008), the standard series expansion of spherical harmonic synthesis is used (e.g., Holmes and Pavlis 2008):

$$\zeta^{EGM2008} = \frac{GM}{r\gamma} \sum_{n=2}^{n_{\max}^{EGM}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{\delta C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\cos\theta) \quad (1)$$

with n degree and m order of the harmonic coefficients and $n_{\max}^{EGM}$ indicating the maximum degree of the series expansion (e.g., 2190), GM (geocentric gravitational constant) and a (semi major axis) are the EGM2008 scaling parameters, γ is normal gravity on the surface of the reference ellipsoid, $\overline{P}_{nm}(\cos\theta)$ are the fully-normalised associated Legendre functions. The coordinate triplet (r,θ,λ) denotes the geocentric polar coordinates of radius, geocentric co-latitude and longitude, which are computed from the geodetic coordinates (φ,λ,h) (geodetic latitude, longitude and ellipsoidal height) for each computation point (Torge 2001, Jekeli 2006). The term $\overline{\delta C}_{nm}$ denotes that the even zonal harmonics of the GRS80 ellipsoid are removed from the EGM2008 even zonal coefficients $\overline{C}_{nm}$, for details see e.g. Smith (1998). The zero- and first degree terms neglected in Eq. (1) are discussed in Sect. 5. Equation (1) can be evaluated with the harmonic_synth software (Holmes and Pavlis 2008).

The use of the spherical harmonic series expansion (Eq. 1) poses the problem of signal omission because of the truncation at the maximum expansion degree $n_{\max}^{EGM}$ (e.g., Gruber 2009). According to Torge (2001, p. 74), a GGM series expansion to $n_{\max}^{EGM}$ may imply fine structures with wavelengths λ of

$$\lambda = 360/n_{\max}^{EGM} \tag{2}$$

or, equivalently, offers a spatial resolution Δx of

$$\Delta x = 180/n_{\max}^{EGM}\,. \tag{3}$$

Evaluating EGM2008 to $n_{\max}^{EGM} = 2190$, the minimum wavelengths λ are about 10 arc minutes (~18 km), which equates to a spatial resolution Δx of 5 arc minutes (~9 km). As such, any gravity field structures at scales shorter than 5 arc minutes are not represented by the EGM2008 degree-2190 series expansion. The omission error is crucial in Alpine terrain, where many mountain-valley structures occur at scales below or just below the EGM2008 resolution. The height anomalies from a degree-2190 expansion may be affected by signal omission errors of several cm up to the dm-order, which is shown later.

3 RTM height anomalies

3.1 Methodology

The RTM technique (cf. Forsberg and Tscherning 1981; Forsberg 1984, 1985) is capable of modelling major parts of the EGM2008 signal omission error (cf. Hirt 2010; Hirt et al. 2010). We construct RTM data as the difference between the 3 arc second SRTM elevation model (post processed release vers4.1) (Jarvis et al. 2008; Reuter et al. 2007) and the DTM2006.0 spherical harmonic expansion of Earth's topography made available by the EGM2008 development team (Pavlis et al. 2007).

DTM2006.0 serves as high-pass filter, removing the long-wavelength features from the SRTM data. It comprises about 2.4 million pairs of fully normalised height coefficients $\overline{HC}_{nm}, \overline{HS}_{nm}$ that give $z^{DTM\,2006.0}$ elevations using (Pavlis et al. 2007):

$$z^{DTM\,2006.0}(\theta,\lambda) = \sum_{n=0}^{n_{\max}^{DTM}} \sum_{m=0}^{n} (\overline{HC}_{nm}\cos m\lambda + \overline{HS}_{nm}\sin m\lambda)\overline{P}_{nm}(\cos\theta) \qquad (4)$$

where $n_{\max}^{DTM}$ is the maximum degree of evaluation (2160). Equation (4) is evaluated to the same spherical harmonic degree $n_{\max}^{DTM} = n_{\max}^{EGM}$ used for the computation of EGM2008 height anomalies (Eq. 1). As a consequence, the SRTM/DTM2006.0 RTM data implies gravity field structures exclusively at scales shorter than EGM2008's spatial resolution.

The SRTM/DTM2006.0 RTM data is transformed to ζ^{RTM} height anomalies using the prism-integration forward-modelling method (e.g., Forsberg 1984, Nagy et al. 2000). The residual SRTM/DTM2006.0 elevation $z_{RTM} = z^{SRTM} - z^{DTM\,2006.0}$ of each grid point represents a rectangular prism of constant density ρ_0 for which the gravitational potential V is computed. With the corner coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) of a single prism, the expression for *V* reads (Nagy et al. 2000):

$$V = G\rho_0 \left|\left|\left| xz\ln(z+r) + yz\ln(x+r) + zx\ln(y+r) - \frac{x^2}{2}\tan^{-1}\frac{yz}{xr} - \frac{y^2}{2}\tan^{-1}\frac{zx}{yr} - \frac{z^2}{2}\tan^{-1}\frac{xy}{zr} \right|_{x_1}^{x_2}\right|_{y_1}^{y_2}\right|_{z_1}^{z_2} \quad (5)$$

where *r* is the distance between the point (*x*,*y*,*z*) and the origin of the coordinate system, and *G* is the gravitational constant. In order to evaluate Eq. (5), the variables (*x*,*y*,*z*) are substituted by the limits $(x_1, y_1, z_1, x_2, y_2, z_2)$, giving a total of 48 terms per prism (Nagy et al. 2000). The standard topographic mass-density ρ_0 of 2670 kg m^{-3} was used. We use $z_1 = 0$ and $z_2 = z_{RTM}$, so that the prism heights $z_2 - z_1$ represent the residual elevations. Equation (5) is based on a planar approximation (Nagy et al. 2000), however, the effect of Earth curvature is taken into account here by a vertical shift of the prism as a function of the distance between each prism and the RTM computation point (cf. Forsberg 1984, p. 111).

For the conversion of the prism's potential *V* to its height anomaly contribution ζ^{prism}, a variant of Bruns's equation is applied (Heiskanen and Moritz 1967, p. 293):

$$\zeta^{prism} = \frac{V}{\gamma_Q} \quad (6)$$

where γ_Q is normal gravity on the quasigeoid. The height anomaly contribution ζ^{RTM} of all prisms forming the RTM is then obtained as sum of the height anomalies ζ^{prism} implied by all the single prisms:

$$\zeta^{RTM} = \sum_{i=1}^{k} \zeta^{prism}(i) \quad (7)$$

with k denoting the number of prisms within some radius R around the computation point. As RTM height anomalies ζ^{RTM} possess spectral power beyond the maximum degree of EGM2008, they represent our estimates of the EGM2008 height anomaly omission error.

We acknowledge that the RTM height anomalies do not rigorously augment the EGM2008 spectral content. This is because the gravitational potential of the topography is a nonlinear function of the height, as can be seen form a spherical harmonic representation of Newton's integral (e.g., Rummel et al. 1988, Ramillien 2002, Kuhn and Featherstone 2003, Kuhn and Seitz 2005). In the case of our RTM ‘corrections’, this effect is implicitly contained as DTM2006.0 is used as a spherical-harmonic reference surface. The approximation error when assuming a linear relationship between topographic height and gravitational potential is estimated to be below 10% of the RTM quasigeoid heights (estimate based on degree variances of the topography-induced potential coefficients, Kuhn 2010, pers. comm.). Given that the amplitudes of the RTM quasigeoid heights are dm order, we consider the approximation error acceptable for our study. A detailed investigation of the approximation error and its reduction remains as a future task.

3.2 Integration radius and RTM grid layout

For a test computation point at “Zugspitze” (Germany’s highest mountain with a height of 2962 m above local mean sea level), Fig. 1 provides insight how the single RTM prisms contribute to RTM height anomaly ζ^{RTM}. The near zone (to 20-40 km distance) generates the largest RTM height anomaly contributions. The height anomaly contribution ζ^{RTM} of the complete RTM is shown as a function of the integration radius R in Fig. 2. Oscillating RTM elevations – reflecting the typical Alpine mountain-valley patterns – directly propagate into the ζ^{RTM} values with amplitudes of 5-15 mm at wavelengths of 10-15 km. To obtain reasonably stable values with remaining convergence errors on the level of few mm, the numerical integration of RTM effects should be carried out at least to R = ~200 km.

In order to reduce computation time, it is common practice to work with high-resolution inner zones and coarser outer zones (e.g., Forsberg 1984, Marti 1997, Hirt and Flury 2008). Here, we use the full SRTM resolution (3 arc seconds, corresponding to 90 x 60 m prisms) for constructing RTM data only within the inner zone around the computation point out to a radius R of 40 km. For the outer zone (40 km ≤ R ≤ 200 km), a coarse RTM (SRTM/DTM2006.0) grid of 15 arc seconds (450 m x 300 m prisms with mean elevations originating from 5 x 5 averaged SRTM 3 arc second cells) was used.

3.3 Role of topographic mass-density anomalies

The RTM approach is capable of delivering the quasigeoid contribution ζ^{RTM} beyond degree $n_{\max}^{DTM}$, as generated by a *model topography* of homogeneous density ρ_0. Naturally, such a technique only approximates the actual quasigeoid contribution originating from the *real topography*. This is because the real topography is subject to mass-density anomalies (cf. Forsberg 1984), i.e. disturbing bodies with a contrasting, mostly lower, density ρ with respect to the standard rock density ρ_0 of 2670 kg m^{-3}:

$$\Delta\rho = \rho - \rho_0 \qquad (8)$$

For example, Alpine valleys with Pleistocene sedimentary fillings to a depth of several 100 m may exhibit a density anomaly $\Delta\rho$ of up to -500 kg m^{-3} (Flury 2002). Other, frequently occurring density anomalies in mountainous regions are lakes ($\Delta\rho$ of about -1700 kg m^{-3}) and glaciers ($\Delta\rho$ of roughly -1800 kg m^{-3} or larger, depending on the condition of the ice). Provided that sufficient information on the geometry and density of such anomalies is available, explicit consideration in gravity field modelling in general (e.g., Marti 1997) and in the RTM approach is possible, thus improving the accuracy of the RTM height anomalies.

While the Molodensky theory for the quasigeoid avoids the use of topographic mass-density information, it still should be used in the RTM approach because mass anomalies affect gravity. This is, however, not done here as detailed and extensive knowledge of such local mass-density anomalies is not available. Instead, simulations were carried out in order to assess the maximum contribution of typical Alpine density anomalies on the RTM quasigeoid undulations. Importantly, the dimensions of the simulated disturbing bodies need not exceed EGM2008's spatial resolution of about 10 km. Gravity field features at larger scales are assumed to be – at least formally – represented by EGM2008, and are consequently not the subject of omission error modelling.

In our simulation, we approximated the lakes, glaciers and valley fillings by rectangular prisms of different size and computed the quasigeoid contribution (Eq. 3) at the centre of the upper prism surface, where the quasigeoid effect is maximum. From Table 1, the maximum quasigeoid contribution $\Delta\zeta$ of water bodies is about 5 cm in extreme cases, and those of the Pleistocene valley fillings is about 4 cm.

In practice, however, the spatial dimensions of such anomalies will mostly be smaller, and likewise for the generated quasigeoid contribution. Further to this, the amplitudes of quasigeoid effects always attenuate with increasing distance of the computation point from the disturbing body. Accounting for these unmodelled effects originating from density anomalies, for the approximate character of the RTM corrections (cf. Sect. 3.1), and for the convergence error of a few mm (Sect. 3.2), we conclude that a reasonable accuracy estimate for the RTM height anomalies is 1-2 cm.

4. Test area and comparison data

We selected the mountainous German Alps (South-Eastern part of Germany) with elevations ranging from 500 m to 3000 m (Fig. 3) as our test area. This was chosen because a reasonably precise national gravimetric quasigeoid model (German Combined Quasigeoid GCG05; BKG 2005, Liebsch et al. 2006) was available for comparison. This allows us to compare the 'traditional' way of computing a regional quasigeoid model (variant 1) and the RTM technique (variant 2).

GCG05 is the result of two independent quasigeoid computations carried out at the Institut für Erdmessung (Leibniz University of Hanover) and the Bundesamt für Kartographie and Geodäsie (BKG). The accuracy specification for the GCG05 quasigeoid undulations is 1-2 cm (BKG 2005, Liebsch et al. 2006).

The GCG05 accuracy specification was checked through external validation with astronomical-topographic levelling (Hirt and Flury 2008), giving very precise differences of height anomalies along profiles. The comparisons carried out in two rugged test areas Harz Mountains (Northern Germany) and Isar Valley (German Alps, cf. Fig.3) showed an agreement of better than 1 cm among GCG05 and the astrogeodetic height anomalies differences over 65 km and 23 km, respectively (Hirt et al. 2007, 2008).

A further astrogeodetic validation experiment along a North-South traverse in Germany (Voigt et al. 2007) revealed an agreement with GCG05 height differences on the 1-2 cm level over the Bavarian parts of our test area (Fig. 9 in Voigt et al. 2007). As such, a fairly good quality indicated by these independent comparisons makes GCG05 suited to serve as a reference for the evaluation of the EGM2008/RTM quasigeoid computation approach. It should be noted that the astrogeodetic validation experiments provided a check on quasigeoid height differences and not on (absolute) quasigeoid heights (e.g., Hirt et al. 2007).

We acknowledge firstly that EGM2008 and GCG05 are not independent of each other because – at least partly – similar terrestrial gravity data sets were utilised in both computations. As we do not have access to these gravity data, we cannot rigorously quantify the impact of this effect on our results. However, our numerical results described later provide some evidence that the interdependency has rather low impact on the results of our study. Secondly, the gravity field spectrum is not fully represented by GCG05 because of its 1 arc minute x 1.5 arc minute grid spacing. GCG05, however, offers insight into the EGM2008/RTM solutions over a large area of grid points and not only at few scattered locations, as GPS/levelling data does.

We use the GCG05 quasigeoid undulations within the latitude range 47.2°N to 48.0°N and the longitude range 9.5°E to 13.5°E. This test area includes the South-Eastern German territory of the Alps (Fig. 3). The GCG05 quasigeoid grid was bilinearly interpolated to a higher resolution of 0.005°, giving a total of 87,207 data points. The interpolated GCG05 grid does not provide more information than the original grid, but the RTM field is better resolved at a resolution of 0.005°.

As a second comparison data set, we use a GPS/levelling data set provided by the Bundesamt für Kartographie and Geodäsie (BKG) (Ihde and Sacher 2002). This provides quasigeoid undulations at 34 scattered locations in our South-German test area (cf. Fig. 3). Gruber (2009) already used this set for evaluation of the EGM2008 gravity field model over Germany, however, without RTM omission error corrections as is done in the present study. Importantly, the GPS/levelling data is independent of EGM2008 and contains the full gravity field signal. As such, it circumvents the two drawbacks associated with GCG05 and represents a valuable supplement to our numerical tests using the GCG05 model.

5. Computations

For the comparisons between GCG05, EGM2008 and RTM data, we started by computing estimates of ellipsoidal heights h for the 87,207 GCG05 grid points in our test area. This is achieved by a simple addition of SRTM heights z^{SRTM} (heights above mean sea level) and GCG05 quasigeoid undulations ζ^{GCG05}. Together with geodetic latitude φ and longitude λ, the ellipsoidal height estimates $h = z^{SRTM} + \zeta^{GCG05}$ represent the 3D locations of our computation points. The construction of ellipsoidal heights is similar to that used by Claessens et al. (2008) for EGM2008 evaluation. The zero-tide version of EGM2008 (Pavlis et al. 2008) to degree $n_{\max}^{EGM}$ = 2190 is used along with the harmonic_synth software (Holmes and Pavlis 2008) for evaluation of Eq. (1) at the 87,207 (φ,λ,h) surface points, giving EGM2008 quasigeoid undulations $\zeta^{EGM2008}$.

For the comparisons involving GPS/levelling data, ellipsoidal heights h are immediately available and do not need to be 'constructed' from SRTM elevations and quasigeoid corrections. The importance of evaluating the EGM2008 spherical harmonic coefficients at the 3D locations of the computation points should be stressed here; a simple comparison with evaluations on the ellipsoid ($h=0$) showed an impact of up to 1-2 dm on the EGM2008 quasigeoid undulations in our test area.

The RTM elevations were obtained from SRTM elevations referred to the DTM2006.0 spherical harmonic topography, expanded to spherical harmonic degree $n_{\max}^{DTM}$ = 2160 (Eq. 4). The computation of the RTM height anomalies ζ^{RTM} (Eqs. 5-7) was performed for each of the 87,207 grid points, based on evaluating the SRTM/DTM2006.0 residual elevations within grids of 40 km and 200 km radii, respectively, using software based on the TC program (Forsberg 1984). Finally, the RTM height anomalies were algebraically added to the EGM2008 height anomalies, giving EGM2008/RTM quasigeoid undulations $\zeta^{EGM2008/RTM}$.

In our study, we follow the 'official' recommendation of EGM Development Team (2008) to use EGM2008 to degree $n_{\max}^{EGM}$ = 2190 and the DTM2006.0 spherical harmonic topography to degree $n_{\max}^{DTM}$ = 2160. Further to this, we repeated the computation procedure for a range of maximum spherical harmonic degrees ($n_{\max}^{DTM} = n_{\max}^{EGM} \in$[360, 720, 1080, 1440 and 1800, 2160]), allowing us to test a variety of EGM2008/RTM quasigeoid solutions.

Because many geodetic applications use *differences of quasigeoid heights* rather than absolute values (cf. Featherstone 2001), we applied a 'bias fit' to the differences in any of our comparisons between the EGM2008/RTM quasigeoid solutions and the GCG05 geoid model and GPS/levelling points. This procedure eliminates the impact of neglected zero and first degree terms in Eq. (1) and (constant) vertical datum offsets between the models. This is also consistent with other studies, which make use of standard deviations (STD) from differences rather than RMS values as performance indicator (e.g., Burša et al. 2009; Ågren 2009).

6. Results and Discussion

Table 2 reports the descriptive statistics of the RTM height anomalies from Eq. (7). The RTM (degree 2160) height anomalies possess an average signal power of about 3 cm with maximum values of the order of 15 cm. These values give some indication of the signal omission of EGM2008 (degree 2160) height anomalies in mountainous terrain. As expected, the RTM quasigeoid contributions increase with decreasing degree of the DTM2006.0 reference surface.

Figure 4 shows the key results of our study. It illustrates the comparison between GCG05, EGM2008 (degree 2190) and RTM (degree 2160) height anomalies in our test area.

The differences GCG05-EGM2008 (Fig. 4A) show residual patterns with amplitudes of up to 20 cm and wavelengths mostly of about 10-20 km. 20 km roughly equates to the shortest wavelengths implied by EGM2008 (Eq. 2), while features with 10 km wavelengths are beyond the resolution of EGM2008. The RTM (degree 2160) height anomalies (Fig. 4B) exhibit residual patterns with similar characteristics. A visual comparison between Figs. (4A) and (4B) shows numerous peak structures that are equally present in the RTM field and the GCG05-EGM2008 differences. The strong correlation between both data sets particularly evident in the Berchtesgaden area (latitude 47.6°N, longitude 13.0°E), but also visible in many other parts of SE Germany, such as the Zugspitze region (latitude 47.4°N, longitude 11.0°E) and Oberstdorf (latitude 47.4°N, longitude 10.25°E)

Figure 4C shows the GCG05–EGM2008 differences, with EGM2008 'augmented' by the RTM height anomalies. The comparison with Fig. 4A reveals a considerable improvement when RTM height anomalies are applied as an omission error 'correction' to EGM2008. The RTM height anomalies diminish almost any peak structure with amplitudes of 10-20 cm to the level of 5 cm or less. The descriptive statistics of the GCG05-EGM2008 and GCG05-EGM2008/RTM comparisons in Table 3 show that the standard deviation decreases from 3.7 cm to 1.9 cm. This equates an improvement rate of about 47%. This shows that augmenting EGM2008 with RTM-based omission error estimates gives significantly more accurate quasigeoid heights than EGM2008 alone in rugged terrain.

Formally, the remaining discrepancies shown in Fig. 4C are attributable to three sources of uncertainty: (1) GCG05 errors, (2) any RTM height anomaly errors including the impact of local mass-density anomalies not modelled by the RTM method, and (3) EGM2008 commission errors, i.e. the uncertainty of the model-implied height anomalies. First, we recall that in our test area the GCG05 height anomaly differences were found to be in cm agreement with external astrogeodetic comparison data. Second, the accuracy of RTM height anomalies was assessed to be on the 1-2 cm level, which includes the impact of unmodelled local density anomalies (see above). Reflecting this, the 2 cm STD value provides some evidence that the impact of unmodelled local mass-density anomalies is (on average) fairly small and indicates a reasonable quality of the RTM corrections done here. Also, this comparison proves the very good quality of the EGM2008 and GCG05 models in the German Alps.

A further indicator of the RTM performance is the reduction rate of residual errors *r*, which may expressed as

$$r = \text{abs}(\zeta^{GCG05} - \zeta^{EGM2008}) - \text{abs}(\zeta^{GCG05} - \zeta^{EGM2008/RTM}) \quad (9)$$

Figure 4D shows this indicator for the 87,207 points. In comparison with Fig. 3, the application of RTM corrections improves the agreement between EGM2008 and GCG05 in most mountainous parts of our test area. A minor deterioration (i.e., negative *r* values), is visible only for small parts of the test area. The reduction rate *r* is strongly related to the presence of Alpine mountain-valley patterns, with improvements frequently found on the 10 cm level.

The results of the comparisons using the 34 GPS/levelling points are reported in Table 4 and shown in Fig. 5. The STD (from GPS/levelling–EGM2008 quasigeoid height differences) is 4.1 cm. Adding our RTM corrections to the EGM2008 height anomalies reduces this value to 2.1 cm, which equates a 49% improvement. Due to the independence of the GPS/lev data from EGM2008, this result is a strong corroboration of our findings on RTM corrections using GCG05 as a reference model. It also shows that the previously mentioned interdependency between GCG05 and EGM2008 (cf. Section 4) plays a rather small role in the present study.

Additional experimental computations were done to investigate whether it is possible to further reduce the difference patterns seen in Fig. 4C by using different spectral degrees for combining EGM2008 with RTM data. For example, with a spherical harmonic degree of 1800, RTM data serves not only as source for estimating the omission error beyond degree 2160 but also for substituting EGM2008's spherical harmonic band 1801 to 2160. We have tested spherical harmonic degrees $n_{\max}^{DTM} = n_{\max}^{EGM}$ = 2160, 1800, 1440, 1080, 720 and 360 to combine EGM2008 with RTM height anomalies.

The descriptive statistics of the comparisons of EGM2008/RTM height anomalies against GCG05 are reported in Table 3 and those against the GPS/levelling data in Table 4. Both sets of comparison data reveal a very similar behaviour of the EGM2008/RTM combinations. For spherical harmonic degrees 1080-2160, slightly increasing residuals are observed with STDs going up from 2.1 cm to 2.6 cm. A significant deterioration in agreement is found for spherical harmonic degrees 720 and in particular, for degree 360 with STDs as high as 7.4 cm.

These results suggest the following for mountainous areas like the German Alps:

(1) The best agreement is observed for EGM2008 used to degree 2190 and RTM omission error corrections from DTM2006.0 expanded to degree 2160, is an empirical endorsement of the 'official' recommendation of the EGM Development Team (2008).

(2) In the high-degree spectral bands of 1081 to 2160, EGM2008 height anomalies are somewhat more accurate than those modelled from RTM data alone.

(3) The deterioration in agreement for degrees 360 and 720 shows that RTM modelling in the spectral range 361 to 1080 is inferior to EGM2008 alone. The impact of unmodelled mass-density anomalies prevails here, hence the need for gravity observations to recover the quasigeoid in this spectral window.

7. Conclusions and Recommendations

This study investigated the RTM method to reduce omission errors of EGM2008 height anomalies in mountainous terrain. The wavelengths of Alpine mountain-valley gravity field structures are often shorter than the EGM2008 maximum degree (2190, corresponding to wavelengths of 10 arc munutes), so are omitted by EGM2008. In our German test area, the EGM2008 only comparisons with the GCG05 model and the GPS/levelling data showed standard deviations of 3.7 cm and 4.1 cm, respectively. Augmentation with RTM omission error estimates reduced these values to 1.9 cm (GCG05) and 2.1 cm (GPS/levelling). These results demonstrate that applying RTM omission error estimates to EGM2008 height anomalies improves the quasigeoid modelling by almost 50 %. As a consequence, the RTM omission error correction applied to EGM2008 is a simple yet effective method to precisely model the quasigeoid in mountainous areas, especially those devoid of gravity data. A further benefit of the proposed approach is the fact that the RTM corrections may be easily computed down to the resolution of the elevation data used.

Our proposed approach to improve EGM2008 with RTM data is not intended to replace high-precision national quasigeoid computations based on Stokes's integral when there is a dense coverage of gravity observations. Instead, we consider our approach to be promising, especially in mountainous regions, where insufficient gravity data coverage impedes precise Stokes-based geoid computation. Potential application areas would be, for example, in Asia, Africa and South America.

In addition, the related computational requirements are low, as only two RTM height anomaly values need to be computed for applications such as GNSS-based height transfer. Importantly, our method may be easily applied – without the need to take any field measurements – using the three free-of-charge data sets (EGM2008, SRTM and DTM2006.0) if the cost of regional gravity surveys is too prohibitive. As a further application, our approach will enable advanced validation of present and future gravity field models, where a reduction of the omission error by means of RTM data allows for a better isolation of model commission errors.

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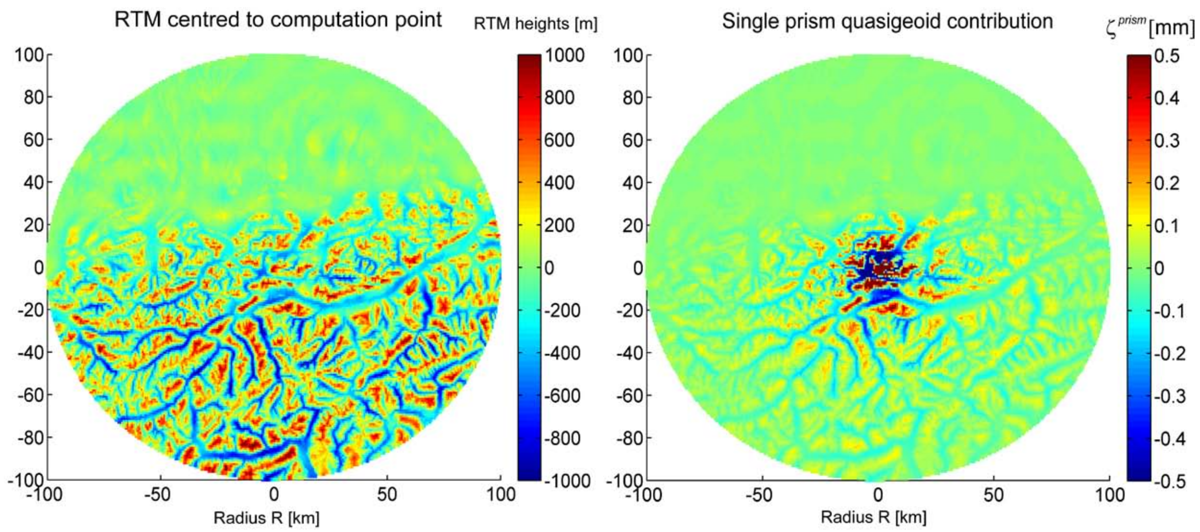

Fig. 1 RTM elevations $z^{SRTM} - z^{DTM\,2006.0}$ [m] (left) and RTM height anomalies ζ^{RTM} [mm] (right) for test computation point Zugspitze (latitude 47.421°N, longitude 10.984°E). For simplicity, a coarser SRTM resolution (30”) has been used in this example.

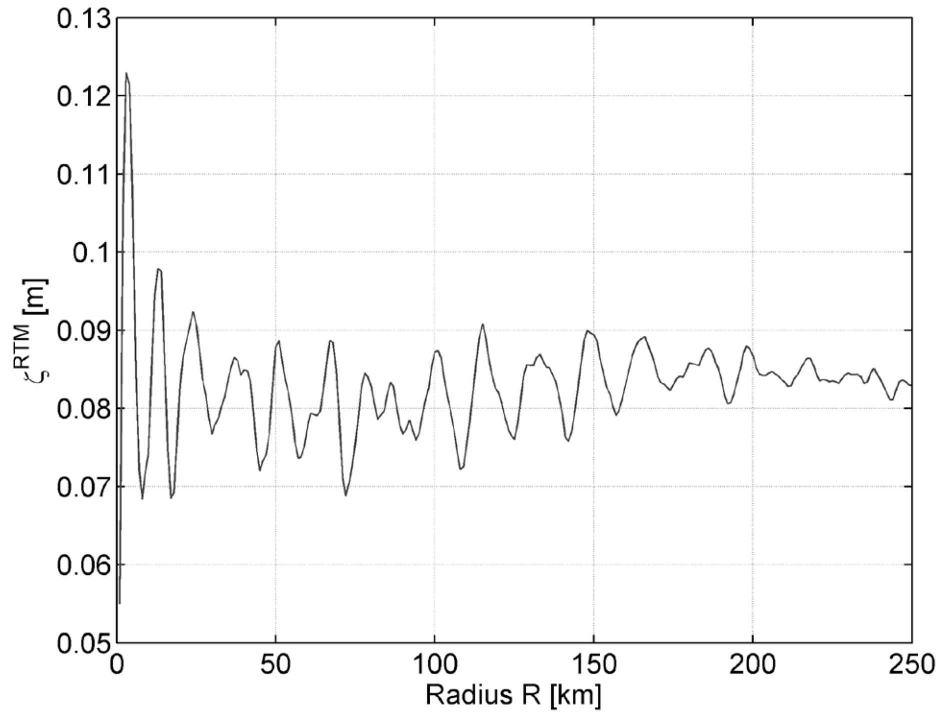

Fig. 2 RTM height anomalies [m] as a function of the computation radius *R* for test computation point Zugspitze.

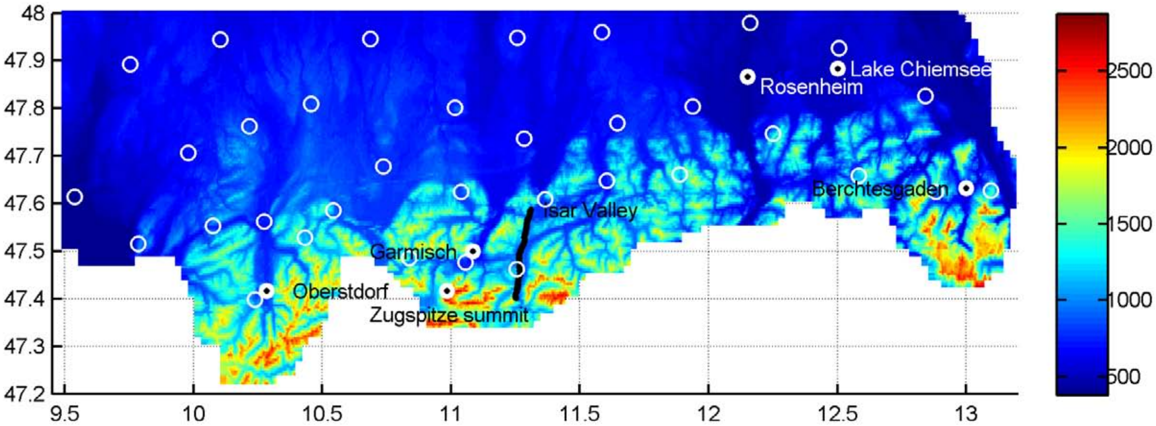

Fig. 3 Test area in the German Alps where GCG05 quasigeoid undulations are available. Topography is from SRTM, heights are in metres. Circles show locations of the GPS/levelling points. Black dots show the astrogeodetic quasigeoid profile used for external GCG05 validation in the Isar Valley.

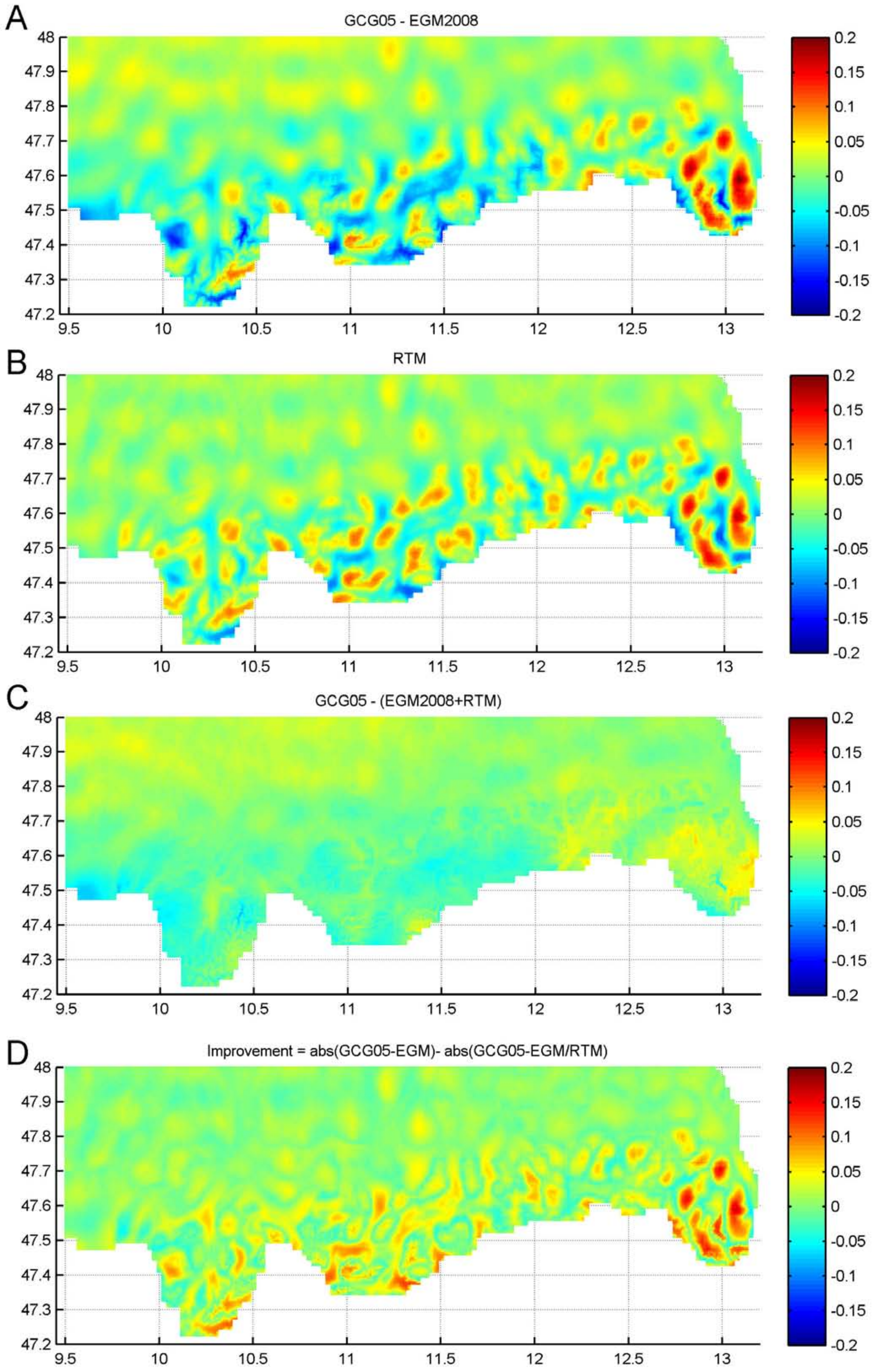

Fig. 4 A: Differences GCG05 – EGM2008 (degree 2190), B: RTM (degree 2160) height anomalies, C: GCG05 – (EGM2008+RTM). D: improvement (reduction rate *r*) of residual errors. Units in metres.

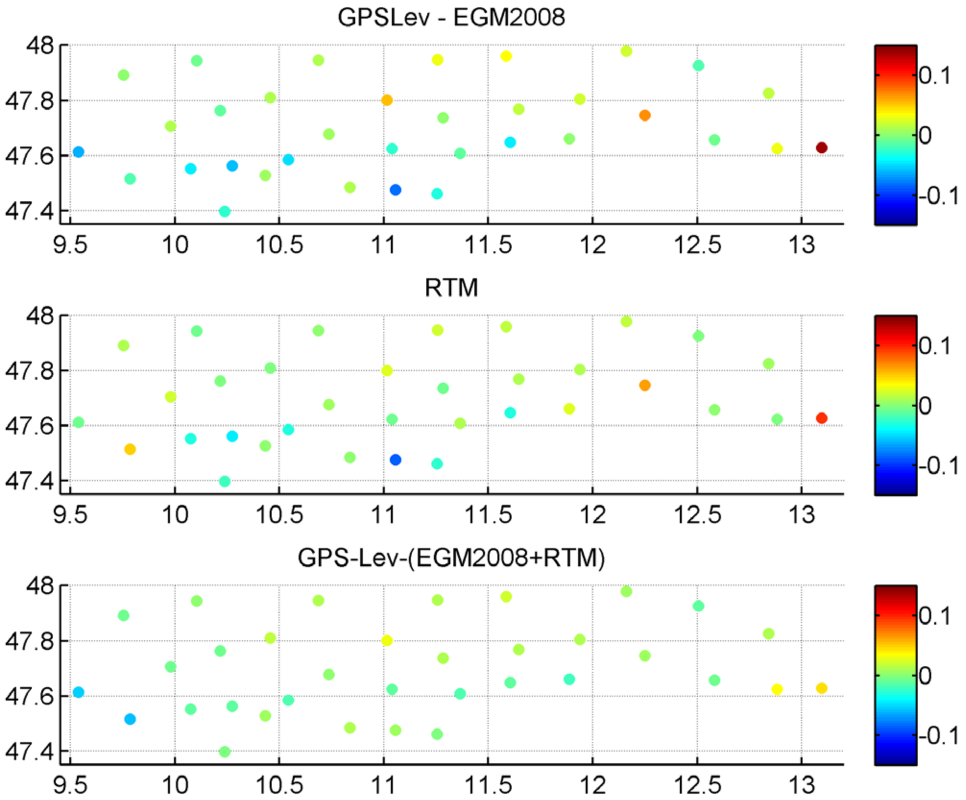

Fig. 5 A: Differences GPS/Lev – EGM2008 (degree 2190), B: RTM (degree 2160) height anomalies, C: GPS/Lev – (EGM2008+RTM). Units in metres.

Table 1 Simulated alpine density anomalies (Δρ, geometry defined by width, length, depth) and their contribution to the quasigeoid Δζ

Simulated body	Density anomaly Δρ [kg m^{-3}]	Width [km]	Length [km]	Depth [km]	Quasigeoid effect Δζ [cm]
Lake or glacier	-1700	1	1	0.1	-0.4
	-1700	1	1	0.2	-0.7
	-1700	4	4	0.1	-1.6
	-1700	4	4	0.2	-3.1
	-1700	10	2	0.2	-2.9
	-1700	10	5	0.2	-5.4
Valley filling	-500	4	2	0.5	-1.4
	-500	10	2	0.5	-2.0
	-500	2	10	0.5	-2.0
	-500	10	5	0.5	-3.8

Table 2 Descriptive statistics of RTM height anomalies at 80,207 points in SE Germany as a function of the spherical harmonic degree of the DTM2006.0 reference surface. Units are metres.

Degree	Min	Max	Mean	STD
2160	-0.132	0.178	0.007	0.032
1800	-0.143	0.183	0.007	0.037
1440	-0.165	0.217	0.008	0.047
1080	-0.217	0.271	0.009	0.056
720	-0.294	0.486	0.010	0.085
360	-0.343	0.442	0.043	0.145

Table 3 Descriptive statistics of comparisons among GCG05 and EGM2008 only, as well as EGM2008/RTM solutions as a function of different spectral combination degrees. Statistics are based on comparisons of height anomalies at 80,207 points in Southern Germany. A bias fit is applied in each of the comparisons, i.e. the mean value is 0.000 m. Improvement rates are given in terms of the STD and refer to the GCG05-EGM2008 (only) comparison. Units in metres.

Comparison	$n_{\max}^{EGM}$	$n_{\max}^{DTM}$	Min	Max	STD	Improvement rate [%]
GCG05-EGM2008 only	2190	-	-0.163	0.197	0.0366	
GCG05-(EGM2008+RTM)	2190	2160	-0.083	0.071	0.0194	47.0
GCG05-(EGM2008+RTM)	2160	2160	-0.076	0.078	0.0198	46.0
GCG05-(EGM2008+RTM)	1800	1800	-0.072	0.071	0.0206	43.8
GCG05-(EGM2008+RTM)	1440	1440	-0.076	0.064	0.0208	43.1
GCG05-(EGM2008+RTM)	1080	1080	-0.080	0.081	0.0241	34.2
GCG05-(EGM2008+RTM)	720	720	-0.109	0.083	0.0293	19.9
GCG05-(EGM2008+RTM)	360	360	-0.145	0.256	0.0781	-113.4

Table 4 Descriptive statistics of comparisons among GPS/lev and EGM2008 only, as well as EGM2008/RTM solutions as a function of different spectral combination degrees. Statistics are based on comparisons of height anomalies at 34 points in Southern Germany. A bias fit is applied to each of the comparisons. Improvement rates are given in terms of the STD and refer to the GPS/lev-EGM2008 (only) comparison. Units in metres.

Comparison	n_{max}^{EGM}	n_{max}^{DTM}	Min	Max	STD	Improvement rate [%]
GPS/lev -EGM2008 only	2190	-	-0.083	0.137	0.0405	
GPS/lev -(EGM2008+RTM)	2190	2160	-0.061	0.045	0.0205	49.4
GPS/lev -(EGM2008+RTM)	2160	2160	-0.061	0.040	0.0214	47.0
GPS/lev -(EGM2008+RTM)	1800	1800	-0.058	0.053	0.0251	38.0
GPS/lev -(EGM2008+RTM)	1440	1440	-0.065	0.041	0.0226	44.2
GPS/lev -(EGM2008+RTM)	1080	1080	-0.063	0.058	0.0262	35.2
GPS/lev -(EGM2008+RTM)	720	720	-0.100	0.049	0.0338	16.6
GPS/lev -(EGM2008+RTM)	360	360	-0.130	0.189	0.0741	-82.9