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Astrogeodetic Validation of Gravimetric Quasigeoid Models in the German Alps - First Results

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Abstract. In regions with a rough topography, e.g. the European Alps, the accuracy of geoid or quasigeoid models is often reduced. For the validation and accuracy assessment of gravimetric models, astronomical levelling is a well-suited independent method. In a test area, located in the German Alps, a new astrogeodetic data set was acquired using the Hannover Digital Zenith Camera System. Vertical deflections were determined at 100 new stations (spacing about 230 m) arranged in a profile of 23 km length. Repeated observations at 38 stations in different nights reveal an observational accuracy of about $0\overset{''}{.}08$. In order to precisely interpolate the vertical deflection data between adjacent stations, topographic reductions of the observed deflections are carried out using a high-resolution digital terrain model. A least squares prediction approach is applied for the interpolation of a dense profile of deflection data. Eventually, the topography effect is restored. By computing the normal correction, the deflection data is reduced to the quasigeoid domain. The accuracy of the computed astrogeodetic quasigeoid profile is estimated to be at the millimeter-level. The available quasigeoid models, namely the German Combined Geoid GCG2005, the Digital Finite Height Reference Surface DFHRS and the quasigeoid by IAPG (TU Munich), are in agreement with the high-precision astrogeodetic quasigeoid profile by about 8 mm, 20 mm and 4 mm (RMS), respectively. A comparison of the astrogeodetic profile with GPS/levelling data yielded differences of 10 mm.

Keywords. Digital Zenith Camera System, vertical deflection, astrogeodetic quasigeoid profile, local quasigeoid evaluation

1 Introduction

During the recent years, considerable advances have been made in the astrogeodetic determination of the gravity field with Digital Zenith Camera Systems (Hirt 2004, Hirt and Bürki 2002). These new measurement systems provide vertical deflection data accurate to $0\overset{''}{.}08$-$0\overset{''}{.}1$ at a typical observation time of about 20 min per station. Besides regional applications, e.g. the combined gravity field determination in mountainous areas (e.g. Brockmann et al. 2004), vertical deflections may be used in the method of astronomical levelling in order to determine local geoid and quasigeoid (QG) profiles. Astrogeodetic vertical deflections represent independent observables which can be used for comparison with gravity field models based on gravimetric computation techniques. Provided that vertical deflection data is precisely observed at densely distributed stations and the interpolation between the observation sites is done with sufficient accuracy, astronomical levelling provides the shape of the local gravity field with an accuracy at the millimeter level over distances of about 10-20 km (section 4). As a consequence, astrogeodetic gravity field profiles may be used for the local validation and accuracy assessment of gravimetric gravity field models.

The aim of this work is the validation of different gravimetric QG models by a new set of astrogeodetic vertical deflections. The astrogeodetic data was determined in a test area, located in the German Alps, using the Hannover Digital Zenith Camera System TZK2-D (section 2). Due to the rough Alpine topography, the location is considered to represent a kind of area where gravity field models tend to show a reduced precision (e.g. Denker et al. 2003). The main focus of the paper is put on the thorough computation of the astrogeodetic QG profile. Different aspects are covered such as the role of Digital Terrain Model (DTM) data for topographic reductions, interpolation of the observed deflection data and the transition from the observations to the QG applying the normal correction (section 3). The computed astrogeodetic profile is suited for comparison with GPS/levelling data and gravimetric gravity field models (section 5). In order to avoid any dependencies of the results on density hypotheses, the comparison is restricted to the QG domain.

2 Astrogeodetic Observations

In autumn 2005, the Digital Zenith Camera System TZK2-D was used for extensive vertical deflection measurements at 103 new stations which are arranged in a profile. It is oriented in good approximation in North-South direction. Located in the Isar valley near the Ester mountains, the profile starts at the lake Walchensee, crosses Mittenwald and ends near the German-Austrian borderline. The profile length is about 23.3 km and the average station spacing is approximately 230 m.

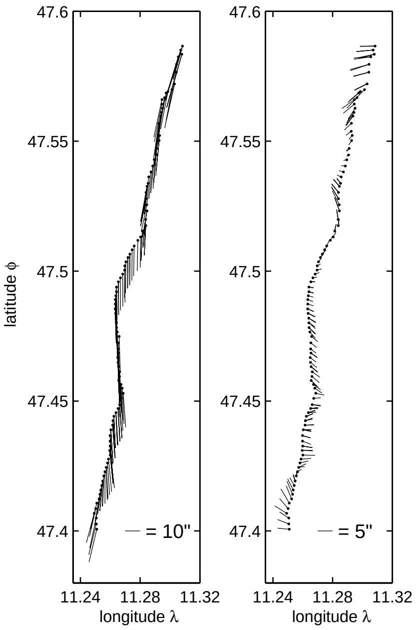

Figure 1. Observed vertical deflection data (left: original data, right: data centered to mean values)

The collection of the vertical deflection data was completed during a total observation period of 4 weeks. The observed data sets were processed using the Hannover astrogeodetic processing system AURIGA (Hirt 2004). The celestial reference was provided by the new high-precision UCAC and Tycho-2 star catalogues (for details see e.g. Zacharias et al. 2000). The campaign and processing statistics are given in Table 1.

Due to the good weather conditions during the campaign, about 38 stations were observed twice in different nights. The standard deviation obtained from the differences is found to be $0\overset{''}{.}082$ both for ξ and η. These accuracy estimates agree well with values from other astrogeodetic measurement campaigns with the same instrument, cf. Hirt and Seeber (2005) or Hirt (2006). The distribution of TZK2-D stations and the acquired (ξ, η)-data is shown in Fig. 1 in vector representation. The vertical deflection field (left part) is obviously dominated by a North-South trend showing the strong gravitational influence of the masses of the central Alps located South of the profile. The right part illustrates the structure of the observations after centering to their mean values. Thereby the largest portion of the attraction of the central Alps is removed and the gravitational attraction of the local topography becomes visible, illustrating the ability of the high-precision measurement system TZK2-D for observation of the fine structure of the gravity field.

Station count	103
Double occupations (in different nights)	38
Station count per night	5-17
Single observations (total)	6700
Single observations (per station)	48
Processed UCAC stars (total)	589000
Processed UCAC stars (per station)	4180

Table 1. Statistics of the astrogeodetic measurement campaign 2005 in the German Alps

3 Astrogeodetic QG Computation

The basic principle of astronomical levelling is to integrate vertical deflections (ξ, η) along a path from station 1 to station n (cf. Torge 2001):

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \qquad (1)$$

$$\Delta\zeta_{1n} = -\int_{1}^{n-1} \frac{\varepsilon_i + \varepsilon_{i+1}}{2} ds_{i,i+1} - E_{1n}^{N} \qquad (2)$$

where ε is the deflection component given in the azimuth α of the section ds between adjacent stations. The term E_{1n}^{N}, referred to as normal correction or normal height reduction, reduces the vertical deflection data to the QG with the result that QG height differences $\Delta\zeta_{1n}$ are obtained. Evaluating the integral given in Eq. 2 presupposes a dense coverage of vertical deflection stations along the path so that the deflection data may be interpolated linearly – with sufficient accuracy – between adjacent stations (cf. Torge 2001). Such a dense coverage is particularly important in case of rough topography.

3.1 Interpolation of Deflection Data

The variation of observed vertical deflections $(\xi, \eta)_{obs}$ originates to a large extent from the gravitational forces of the local topographic masses (cf. Fig 1). DTM data may be used for the computation of topographic vertical deflections $(\xi, \eta)_{top}$, e.g. by applying the prism method (cf. Forsberg and Tscherning 1981, Denker 1988, Flury 2002). A topographically reduced set of vertical deflections shows a much smoother behaviour than the observed surface data. It is suited for interpolation of deflection data $(\xi, \eta)_{prd}$ at intermediate stations, applying techniques such as least squares prediction.

For the topographic reduction of the observed deflection data $(\xi, \eta)_{obs}$ a local high-resolution DTM (spatial resolution of 50 m, area coverage of 50 km x 60 km) was provided by the surveying authority of the state Bavaria. It was used for the computation of a set of topographic vertical deflections $(\xi, \eta)_{top}$ at the TZK2-D stations and, in addition, at 9 intermediate points between each pair of observed stations, yielding an average station spacing of about 23 m. A comparison between the least squares interpolation and a simple linear interpolation of surface deflections (without using DTM data) yielded a QG difference of about 1 mm over a distance of 1 km at the beginning of the profile where the topography is extremely rugged. Therefore the simple linear interpolation approach does not meet the accuracy requirements of this work.

Fig. 2 (a) exemplarily shows the topographic deflection component ξ_{top} as well as the ξ_{obs} data derived from the TZK2-D observations for a part of the profile[1]. Note that both data sets show a high-degree of correlation, reflecting the sensitivity of the astrogeodetic observations for the attraction of the local topographic masses. Fig. 2 (b) illustrates the very smooth behaviour of the topographically reduced deflection $\Delta\xi$ after removing the topographic effect from the observations. Figure 2 (c) shows the same quantity $\Delta\xi$, plotted however at a larger vertical scale. The topographically reduced deflection $\Delta\xi$ serves as input data set for the least squares prediction approach that decomposes the reduced deflection $\Delta\xi$ into a filtered component $\Delta\xi_{fil}$ and a residual noise vector ε_{ξ}. The residual noise vector (Fig. 2 (d)) contains random errors of the astrogeodetic observations and uncertainties attributable to the DTM data. The standard deviation computed from the noise vector is found to be $0''.085$ for ξ and $0''.082$ for η. It is considered to be a further confirmation of the high accuracy of the astrogeodetic observations presented in this paper.

For the set of intermediate points (about 900), the described interpolation approach provides predicted values $\Delta\xi_{prd}$. In the last step the topographic effect is restored. The obtained dense data set of predicted vertical deflections $(\xi, \eta)_{prd}$ shows a linear behaviour between each pair of adjacent stations (cf. Fig. 2 (e)). It is suited for integration along the path using the basic equation 2. For a detailed study on the combination of high-precision vertical deflection data and DTM data the reader is referred to Hirt and Flury (2006).

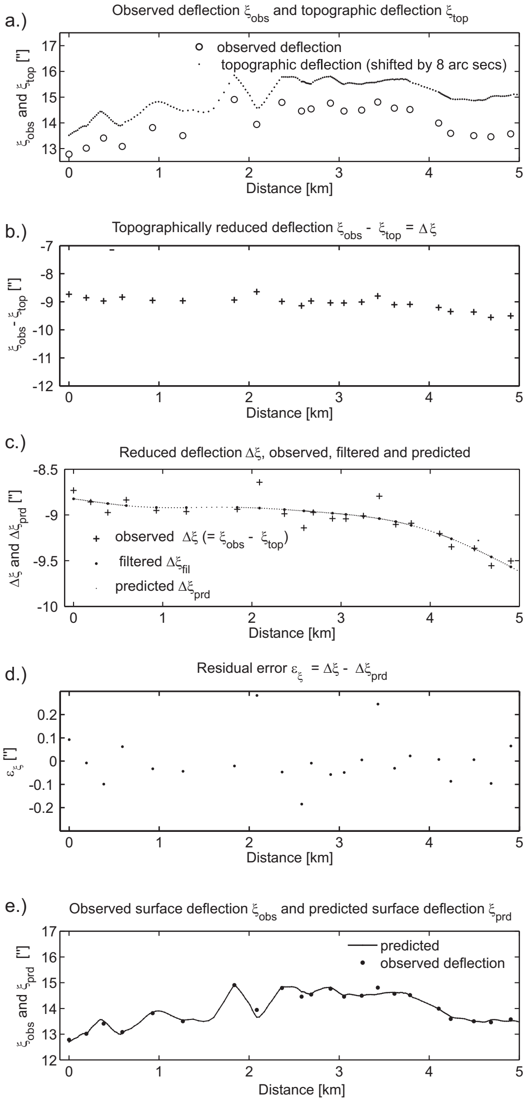

Figure 2. Least squares interpolation approach. (a): observed deflection ξ_{obs} and topographic deflection ξ_{top}. The latter is shifted by $8''$ for better visualization. (b): reduced deflection $\Delta\xi = \xi_{obs} - \xi_{top}$ (after removing the topographic influence from the observations). (c): reduced deflection ξ and predicted values at intermediate points. (d): noise vector ε_{ξ}. (e): result of the restitution: a dense profile of predicted surface deflection data $\xi_{prd} = \Delta\xi_{prd} + \xi_{top}$. Note that the peaks, e.g. apparent in (a) and (e) at distances 0.4 km, 1 km or 1.8 km, originate not from density anomalies but from azimuthal changes in the integration path. In astronomical levelling, peak-like structures are typical features when the stations are not exactly arranged in a straight line.

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[1] Due to the restricted space, the prediction results for the component η are not depicted. They are found in Hirt and Flury (2006).

3.2 Normal Correction

The normal correction E_{1n}^N, which is also known from geometric levelling, is applied for the rigorous reduction of the vertical deflection data to the QG (cf. Torge 2001, p. 251):

$$E_{1n}^N = \int_1^n \frac{g-\gamma_0^{45}}{\gamma_0^{45}} dn + \frac{\overline{\gamma}_1 - \gamma_0^{45}}{\gamma_0^{45}} H_1 - \frac{\overline{\gamma}_n - \gamma_0^{45}}{\gamma_0^{45}} H_n. \tag{3}$$

The computation of the normal correction E_{1n}^N requires the knowledge of the surface gravity g along the profile, the height above mean sea level of the first station H_1 and last station H_n and the height differences dn between adjacent stations. The heights H_1,H_n and dn may be derived from DTM data. The mean normal gravity $\overline{\gamma}_1$, $\overline{\gamma}_n$ at the profile's first and last station as well as γ_0^{45} (arbitrary constant value) are computed using standard formulae of the normal gravity field (cf. Torge 2001, p. 106 and 112).

Today, the surface gravity g may be conveniently derived from gravity databases because the corresponding prediction accuracy of a few mgal meets already the requirements as shown below, and gravimetric measurements would imply additional expenses. Two different databases were used for providing the surface gravity g along the profile. The first one was created at the Physikalisch-Technische Bundesanstalt PTB (Braunschweig, Germany) and is mainly based upon digitized Bouguer anomaly contour maps. The second database is the one of the IAPG (TU Munich) which consists of a very dense set of gravity measurements (density of 2.5 points/km^2) in the test area (cf. Flury 2002). A comparison between the predicted gravity values from both databases with ground truth gravity at 30 stations yielded accuracy estimates of about 2 mgal (PTB) and better than 0.5 mgal (IAPG). Fig. 3 shows the two normal correction profiles E_{1n}^N (PTB) and E_{1n}^N (IAPG), which were independently computed based on gravity predictions from both databases. The difference, depicted in the lower part of Fig. 3, shows that the normal correction is accurate to 0.1-0.15 mm. Hence the accuracy of the predicted gravity is completely sufficient for the QG computation.

3.3 Astrogeodetic QG Profile

Following Eqs. 1-2, the astrogeodetic QG profile is obtained. It is shown in Fig. 4.

4 Accuracy Assessment

Before doing the comparison with the gravity field models it is useful to assess the accuracy of the astrogeodetic QG profile. The observed astrogeodetic data set may be divided into two disjunct subsets in a way that the first set consists of the odd station numbers and the second one of the even stations numbers. Therewith the station spacing of the resulting profiles, each containing 51 stations, is 460 m. The subsets serve as input data for the computation of two independent astrogeodetic QG profiles. The differences give an empirical accuracy estimate of about 1-1.5 mm (cf. Fig. 5). Another assessment method is a formal error estimation based on the error sources affecting the computed QG undulations. Table 2 lists the known error sources as well as their total impact of about 2 mm on the computed QG. The impact of the (ξ, η)- random error on the QG was estimated applying the error propagation law of astronomical levelling, see Hirt and Seeber (2005). The systematic UCAC error is on the order of 0$''$.01 due to Zacharias

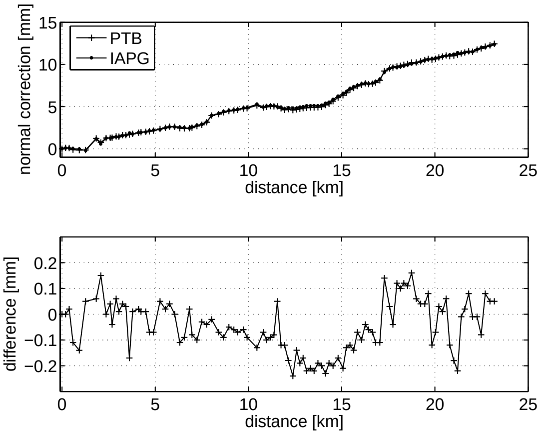

Figure 3. Normal correction of the astrogeodetic profile

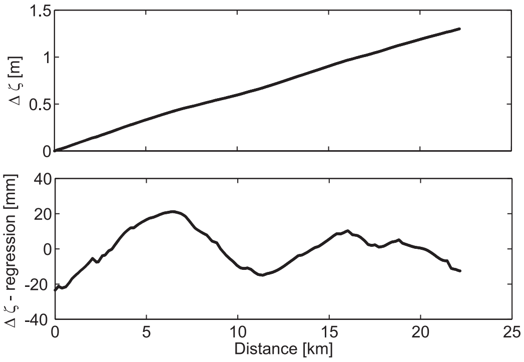

Figure 4. Astrogeodetic quasigeoid profile. The QG height changes by about 1.3 m over a distance of 23 km (upper part). The strong tilt of the QG in southern direction is due to the attraction of the central Alps. Detrending the QG profile makes the fine structure visible (lower part). The small peak-like features are due to azimuthal changes in the integration path.

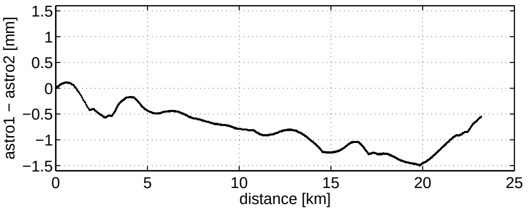

Figure 5. Differences between two quasigeoid profiles computed from two independent data sets of astrogeodetic observations.

et al. (2000). The computed deflection data is assumed to be affected by a systematic error of about $0\farcs005$ since the arithmetic mean from the UCAC and Tycho-2 processing results is used. The uncertainty of the normal correction is derived in section 3.2. It should be noted that another error source in astronomical levelling may be the influence of anomalous refraction on the observed deflection data (ξ,η), see e.g. Hirt (2006). A small remaining refraction error cannot be excluded. However, it is assumed that the largest portion of refraction is cancelled out due to double observations on several stations and the changing weather conditions during the campaign.

The general conclusion is that the astrogeodetic QG computation is accurate to a few millimeters over a distance of 23 km. Therefore it is considered to provide the reference for a comparison with the gravity field models in the next section.

Source / Type	(ξ,η)	$\delta\Delta\zeta$
(ξ,η) random error	$0\farcs08 - 0\farcs09$	0.9-1.1 mm
systematic error from UCAC	$0\farcs005$	0.5 mm
normal correction		0.1-0.15 mm
Total:		$\leq$ 2 mm

Table 2. Estimated error budget for the astrogeodetic QG. The symbol $\delta\Delta\zeta$ refers to the relative error of the QG height difference $\Delta\zeta$ over a profile distance of 23 km.

5 Comparisons

5.1 Astrogeodetic QG vs. GPS/levelling

A first comparison is carried out using a set of 5 GPS/levelling stations, covering the first half section of the astrogeodetic profile. The GPS/levelling data (Flury 2002) provides estimates for absolute QG heights ζ. The RMS computed from the differences between GPS/levelling and the astro-solution (Fig. 6) is 10 mm, and decreases to about 6 mm if the first GPS/levelling station (located eccentrically to the QG profile) is neglected. This very good agreement of the astrogeodetic and GPS/levelling data is at the centimeter accuracy level, normally associated with GPS height measurements.

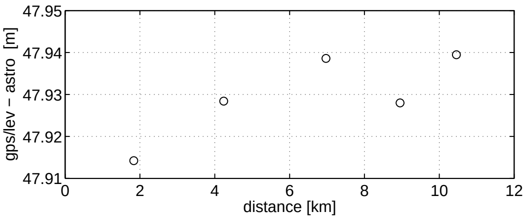

Figure 6. Comparison between GPS/levelling data and the astrogeodetic quasigeoid profile

5.2 Astrogeodetic QG vs. Gravimetric Gravity Field Models

In the working area, three gravimetric gravity field models are available: The German Combined Quasigeoid (GCG) 2005, computed as the average of two independent solutions from the German Federal Agency for Cartography and Geodesy (BKG) and the Institut für Erdmessung (Liebsch et al. 2006). The second model is the digital finite height reference surface DFHRS (Jäger 2006) which is designed as height surface at the 1-3 cm accuracy level. Moreover a gravimetric quasigeoid model for Bavaria, the IAPG QG developed by Gerlach (2003), is also used for a comparison.

The astrogeodetic QG solution does not provide any information on the (absolute) height of the profile. Therefore the comparison is done as bias-fit where the QG height differences at the first station are set to zero. The resulting difference profiles, the main result of this work, are shown in Fig. 7 and the corresponding statistics are listed in Tab. 3. Considering the location of the test area near the German-Austrian borderline and its overall mountainous character (e.g. inhomogeneous and incomplete gravity data), the agreement between the QG models and the astrogeodetic QG is surprisingly good. The GCG2005 agrees with the astrogeodetic QG better than 1 cm (RMS). The RMS difference for the DFHRS amounts to 2 cm, thus remains completely within the associated accuracy specification. An extremely good agreement is found between the IAPG QG and the astrogeodetic QG. Here, the RMS amounts to 4 mm as such reflecting the uncertainties of both data sets. One reason for this excellent result certainly is the much denser set of local input gravity data used in the IAPG QG-computation in comparison to the GCG2005 and DFHRS models.

6 Conclusions

For the astrogeodetic validation of gravity field models, a new high-precision vertical deflection data set

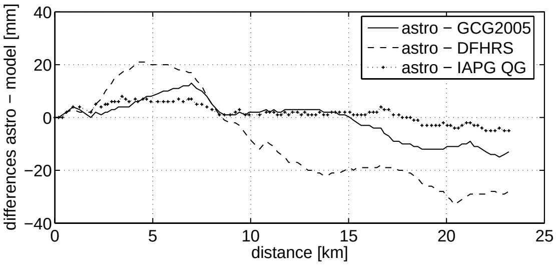

Figure 7. Comparison between the astrogeodetic quasigeoid profile and gravimetric gravity field models

	Astro quasigeoid vs ...		
	GCG [m]	DFHRS [m]	IAPG [m]
Min	-0.015	-0.033	0.005
Max	0.013	0.021	0.008
Mean	-0.001	-0.009	0.002
RMS	0.008	0.020	0.004

Table 3. Statistics of the comparision between the astrogeodetic quasigeoid profile and gravimetric gravity field models GCG, DFHRS and IAPG quasigeoid.

was acquired in the German Alps using the Digital Zenith Camera System TZK2-D. From repeated observations, the noise level of the vertical deflection data is estimated to be about $0\overset{\prime\prime}{.}08$. An independent confirmation, obtained by reducing the observations with DTM data, provides an accuracy estimation for the deflection data of about $0\overset{\prime\prime}{.}09$. The astrogeodetic quasigeoid profile used in the comparison was computed from a combination of the high-precision vertical deflection data, DTM data and predicted surface gravity data. A reasonable accuracy estimate for the astronomical quasigeoid profile is considered to be at the order of a few millimeters over a profile length of 23 km.

The comparison between the astrogeodetic QG and three different gravimetric gravity field models (GCG2005, DFHRS, IAPG QG) reveals a good agreement at the centimeter level. The agreement is considered to be completely satisfactory when taking the mountainous character of the test area into account. An extraordinary good agreement (RMS of 4 mm) is found between the IAPG QG and the astrogeodetic QG. As a general conclusion, this work practically proves the capability of astronomical levelling for the economic determination of quasigeoid profiles with millimeter-accuracy over 10-20 km. To the knowledge of the authors, this is the first time that a consistency at the millimeter level is obtained between an astrogeodetic and gravimetric gravity field model in a mountainous region.

7 Acknowledgements

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