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# Heights in the Bavarian Alps: Mutual Validation of GPS, Levelling, Gravimetric and Astrogeodetic Quasigeoids

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**Abstract.** For many modern regional geoid and quasigeoid models accuracies of one or a few centimeters are given. Validation of these models, however, is often difficult due to the lack of independent data. In a test area of the Bavarian Alps, Germany, with dense data coverage, we carry out validation experiments using GPS, levelling and astrogeodetic observations as well as state-of-the-art regional quasigeoid models. The data cover very rough terrain and mountain peaks. We find a good mutual agreement with a typical scatter of height residuals of 1 – 2 cm. In some areas, however, systematic residual mean values of several centimeters are obtained which are possibly due to residual quasigeoid errors or due to a weak tie to the GPS reference frame.

**Keywords.** Heights, GPS-levelling, quasigeoid, astrogeodetic quasigeoid

## 1 Introduction: Regional Validation of Quasigeoid Models

Many state-of-the-art high resolution geoid and quasigeoid models heavily rely on GPS and levelling data. They are constrained by sets of stations where geoid undulations or height anomalies from GPS and levelling are available. In some cases the GPS-levelling data are directly used as observations applying methods such as least squares collocation or point mass modelling. In other cases, gravimetric models are fitted in an additional step to the GPS-levelling data.

In Germany, both types of approaches have been realized. At BKG, quasigeoid models have been derived from a combination of GPS-levelling and gravity anomaly data using point mass modelling (Ihde et al. 1998). IfE has computed gravimetric quasigeoid models (Denker et al. 2005) which have been fitted to GPS-levelling data using a least squares collocation method. At IAPG, a similar approach has been carried out for Bavaria / South-East Germany (Gerlach 2003). The BKG and IfE solutions have recently been combined to form the national GCG05 quasigeoid model (Liebsch et al. 2006).

The sets of GPS-levelling stations constitute an essential part of the quasigeoid models and of the related vertical reference frame. In between the GPS-levelling stations the quasigeoid models are determined by gravity data and topographic information.

The validation of such high resolution regional quasigeoid models is rather difficult. As the acquisition of precise GPS-levelling data is expensive, often most or all available stations are included in the quasigeoid solution. In consequence, no or few stations remain for validation which makes accuracy estimates for the areas between the GPS-levelling stations uncertain. This holds in particular in mountains, where errors of all involved components – GPS, levelling and geoid/quasigeoid modelling – tend to increase. Furthermore, GPS-levelling data cannot provide a fully independent validation as they are tied to the same GPS and levelling networks as are the stations included in the quasigeoid solution. Note that this situation is different for the validation of satellite derived gravity field models where GPS-levelling data can provide information with a much higher degree of independence.

In a test region in the Bavarian Alps with an extent of 30 km by 80 km, new precise GPS-levelling data are available, including summits up to a height of 2000 m. The area is also densely covered by gravity data (Flury 2002, 2006).

In this paper, we analyze height residuals $\varepsilon$

$$\varepsilon = h_{\mathrm{GPS}} - H_{\mathrm{N}} - \zeta_{\mathrm{Model}}, \qquad (1)$$

where $h_{\mathrm{GPS}}$ is the ellipsoidal GPS height, $H_{\mathrm{N}}$ is the normal height from levelling and gravimetry, and $\zeta_{\mathrm{Model}}$ is the height anomaly from either of the GCG05 and IAPG quasigeoid models. We discuss the suitability of the data sets for validation. In the central part of the area, an exceptional station density is available which allows analyzing not only the magnitude of the residuals but also correlations between stations.

In the same area, new precise deflections of the vertical observed using the Hannover digital zenith camera system TZK2-D are available, in particular along a 23 km dense profile (Hirt et al. 2006, Hirt and Flury 2006). From these data, very precise relative height anomalies have been derived which provide a fully independent quasigeoid validation.

## 2 The GPS and Levelling Data

In the period 1995 – 2000 IAPG carried out precise GPS and levelling measurements for a total of 56 stations in the Estergebirge gravity field test area (Fig. 1, cf. Flury 2002) and in a larger, surrounding area of 30 km by 80 km (Fig. 2). GPS occupation time was 6 – 8 hours per station. For GPS data analysis, several software packages and troposphere modelling approaches have been tested, and accuracies of 1 – 2 cm have been obtained. The station set has recently been tied to the new German SAPOS-ETRS89 GPS reference frame (Klein et al. 2004). This tie has been realised in a single station only.

In the same region, 13 stations are available where BKG carried out 48 hours GPS observations in the year 1999. These stations also have been determined in the SAPOS-ETRS89 reference frame.

All stations have been tied to the first order levelling network DHHN92. Normal levelling reductions have been determined using densely sampled gravity data along the levelling lines. Misclosures of the involved levelling loops are below 1 cm for the Estergebirge area and below 1.7 cm for the larger area. For spatial scales around 20 km and below, the levelling errors are probably smaller than the GPS errors.

Most of the BKG GPS-levelling stations have been included in the GCG05 and IAPG quasigeoid models. This makes these models consistent with the GPS and levelling vertical reference frames. The numerous IAPG stations have not been included and therefore provide a regional validation of these models.

## 3 Analysis of Height Residuals

Height residuals $\varepsilon$ have been determined using Eq. (1) for two station subsets and for both GCG05 and IAPG quasigeoid models. The residuals are shown in Figs. 1 and 2, and the statistics of the residuals are given in Tables 1 and 2. The residuals are the sum of errors in all three components – GPS, levelling and quasigeoid model – which cannot be separated without additional information.

Stations included in the quasigeoid models – marked with triangles in Figs. 1 and 2 – show small residuals. These depend on the weight attributed in the quasigeoid solution to the GPS-levelling data. For the other, independent stations residual standard deviations between 1.2 cm and 1.7 cm have been obtained. The GPS accuracy estimates alone could already explain such residuals. The maximum residuals in the Estergebirge, however, amount up to 7 cm. The mean of the residuals is significant at least for the Estergebirge subset (3.1 cm and 3.7 cm for both quasigeoid models). Most residuals are systematically positive (white columns in the Figures). The rather weak tie to the GPS reference frame may contribute to these systematic residuals, cf. Sect. 2.

Residuals show considerable correlations between neighbouring stations. These may be related to the fact that GPS observations have been carried out under similar conditions (troposphere, reference station constellation) in neighbouring stations. On the other hand, correlations are also expected from levelling errors and quasigeoid model errors (Gerlach 2003). The residuals are similar for both quasigeoid models.

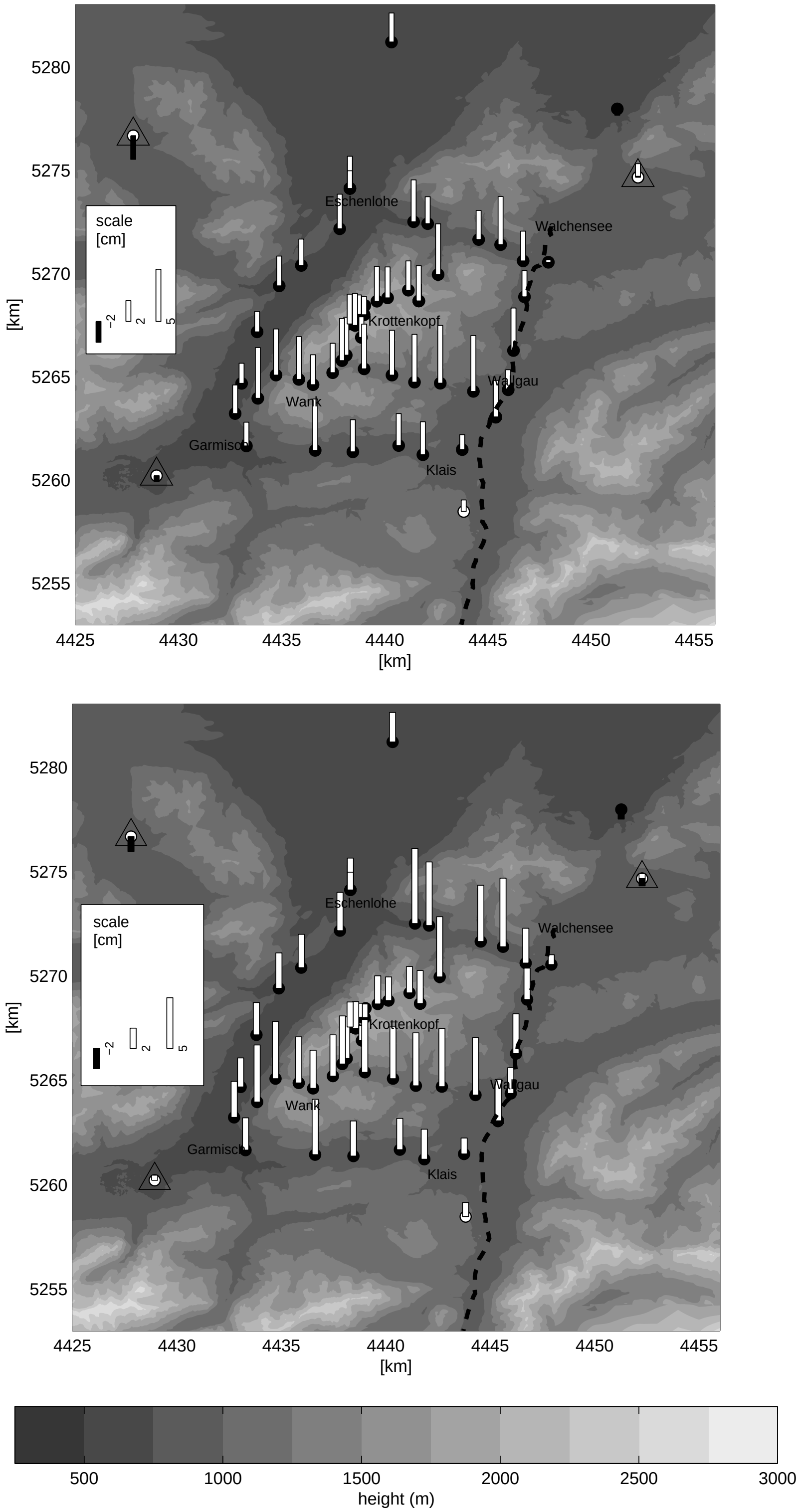


**Fig. 1** Height residuals $\varepsilon$ for Estergebirge area from GPS, levelling and GCG05 quasigeoid model (top) or IAPG quasigeoid model (bottom). Stations used to constrain the quasigeoid models are marked with large triangles. Dashed line: astrogeodetic quasigeoid profile

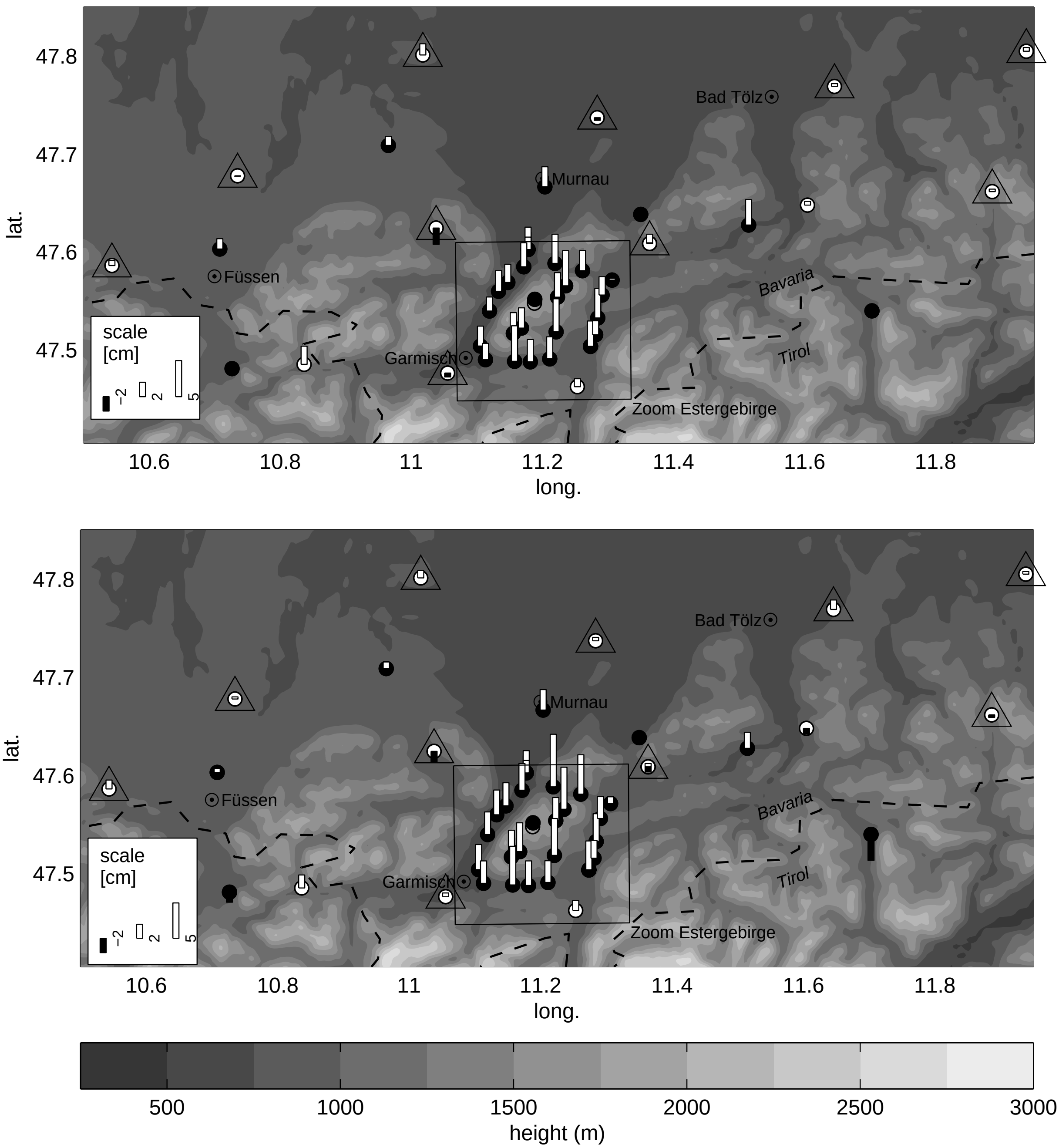


**Fig. 2** Height residuals $\varepsilon$ for larger area from GPS, levelling and GCG05 quasigeoid model (top) or IAPG quasigeoid model (bottom). Stations used to constrain the quasigeoid models are marked with large triangles.

**Table 1** Statistics of height residuals from GPS, leveling and GCG05 quasigeoid, in cm (independent stations only)

| Area | # stations | Min | Max | Mean | Std |
|---|---|---|---|---|---|
| Estergebirge | 50 | -0.4 | 5.5 | 3.1 | 1.2 |
| Larger area | 6 | -0.6 | 3.5 | 1.6 | 1.4 |

**Table 2** Statistics of height residuals from GPS, leveling and IAPG quasigeoid, in cm (independent stations only)

| Area | # stations | Min | Max | Mean | Std |
|---|---|---|---|---|---|
| Estergebirge | 50 | -0.9 | 7.4 | 3.6 | 1.7 |
| Larger area | 6 | -0.9 | 2.9 | 1.0 | 1.4 |

The differences between both quasigeoid models have been found to be 1 cm or less near the constraining GPS-levelling stations. In between of these stations, differences of up to ± 3 cm have been obtained. Close to and beyond the border to Austria no GPS-levelling data have been included in the quasigeoid models which results in differences up to 10 cm between both models. The total quasigeoid range within the test area is about 2 m, with a steep southward increase towards the central Alps.

## 4 Comparison with a Precision Astro-geodetic Quasigeoid Profile

Along a 23 km profile through the Estergebirge area (dashed line in Fig. 1) observations with the Hannover digital zenith camera system have been carried out in autumn 2005. An exceptional station density with a mean station spacing of 230 m and an excellent measurement accuracy of 0.08” rms has been achieved (Hirt et al. 2006). From these data, a profile of relative height anomalies has been determined with an accuracy of better than 2 mm.

This astrogeodetic height anomaly profile provides a reference solution for a fully independent and separate validation of relative height anomalies of the GCG05 and IAPG quasigeoid models, in contrast to the experiment of Sect. 3 where a separation from GPS and levelling errors was not possible. As GPS-levelling data are available in 5 stations along the profile, these can also be validated using the astrogeodetic profile. Fig. 3 shows residuals with respect to the astrogeodetic reference for both quasigeoid models and for the GPS-levelling data. The residuals of the quasigeoid models are set to zero in the first profile station, whereas the residuals of the GPS-levelling data are obtained from a least squares bias fit. The corresponding statistics are given in Table 3.

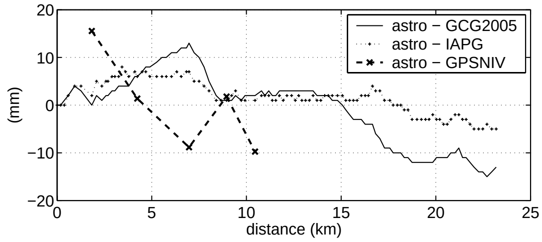


**Fig. 3** Residuals with respect to the astrogeodetic reference solution for both quasigeoid models and for GPS leveling data at 5 stations.

**Table 3** Statistics of residuals with respect to the astrogeodetic quasigeoid solution, in cm.

| Residuals astrogeodetic vs. | Min | Max | Std |
|---|---|---|---|
| GCG05 | -1.5 | 1.3 | 0.75 |
| IAPG | -0.5 | 0.8 | 0.34 |
| GPS-Levelling | -1.0 | 1.6 | 1.02 |

The fit of the IAPG quasigeoid model is excellent with an rms of 3.4 mm. For the GCG05 quasigeoid a 7.5 mm rms is obtained. The misfit of the GPS-levelling data is slightly higher with 1.0 cm, which is in agreement with the residuals of the GPS-levelling data sets analyzed in Sect. 3. At such short scales and for this particular profile, the quality of the astrogeodetic and gravimetric quasigeoid models seems to be better than the quality of the available GPS-levelling data.

## 5 Conclusions

In the Bavarian Alps, height residuals from a comparison of GPS ellipsoidal heights, normal heights from levelling and gravimetry, and regional quasigeoid models have been analyzed. The GCG05 quasigeoid of BKG and IfE and the IAPG quasigeoid have been used in the analysis. The obtained residuals provide a mutual validation only for spatial scales shorter than the distance between the GPS-levelling stations used to constrain the quasigeoid models, which is typically about 15 km. Residual standard deviations of 1 – 2 cm have been found. For the dense Estergebirge data set, a systematic mean of about 3 cm has been obtained. The results are within the accuracy expectations and indicate good quality for all 3 components GPS, levelling and quasigeoid models. This includes very rough mountainous terrain such as sharp peaks. The error contributions of the 3 components cannot be separated without additional information.

An additional validation experiment has been carried out in terms of relative quasigeoid heights, comparing the quasigeoid models and GPS-levelling data to a reference high-precision astrogeodetic zenith camera profile of 23 km length. The obtained residual standard deviations are 3.4 mm for the IAPG quasigeoid and 7.5 mm for the GCG05 quasigeoid. This indicates a very good quality of these models at such spatial scales and shows that with modern models the geoid/quasigeoid error can be smaller than GPS and levelling errors.

For further improvement of such mutual validation experiments, an improved realization of the ellipsoidal GPS heights is important, using longer observation periods and an improved tie to the GPS reference frame. On the other hand, an extension of precise astrogeodetic data would help to separate and understand the individual error contributions. A northward extension of the profile described here is currently carried out connecting existing GPS-levelling stations.

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